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# Effective Development of Reconfigurable Systems Using Linear State-Feedback Control

Scott M. Ferguson* and Kemper Lewis†
*University at Buffalo–State University of New York, Buffalo, New York 14260*

**As the complexity of system design has substantially increased, so, too, has the need for incorporating tradeoffs into the design process. When such tradeoffs are incorporated, the optimal system performance of each objective is sacrificed to increase the overall range of the system's functionality. Reconfigurable systems have been defined as systems designed to maintain a high level of performance through real-time change in their configuration when operating conditions or requirements change in a predictable or unpredictable way. When effectively applied, these reconfigurable systems have the ability to achieve the optimal performance for all system objectives, while also meeting the different constraints of each objective. A method of designing effective reconfigurable systems is presented that focuses on determining how the design variables of a system change, as well as investigating the stability of a reconfigurable system through the application of a state-feedback controller. This method is then applied to a study involving the design of a reconfigurable Formula One race car traversing a predefined racetrack where the design variables change values to achieve maximum performance.**



## Nomenclature

| | | |
|---|---|---|
| $a$ | = | distance from front of vehicle to c.g., ft (m) |
| $a'$ | = | normalized c.g. location |
| $b$ | = | distance from rear of vehicle to c.g., ft (m) |
| $C_{\ell F}$, $C_{\ell R}$ | = | coefficient of lift front and rear |
| $C'$ | = | normalized coefficient of lift |
| $f$, $g$ | = | generic functions |
| $h$ | = | height of c.g., ft (m) |
| $J$ | = | cost function for linear quadratic regulator (LQR) approach |
| $K_F$, $K_R$ | = | roll stiffness front and rear, lb · ft/deg (N · m/deg) |
| $K'$ | = | normalized roll stiffness |
| $L$ | = | length of straightaway, ft (m) |
| $\ell$ | = | vehicle wheelbase, ft (m) |
| $R_1$, $R_2$ | = | radius of corner, ft (m) |
| $S$, $Q$, $R$ | = | weight matrices for LQR approach |
| $u$ | = | longitudinal velocity, ft/s (m/s) |
| $V$ | = | vehicle velocity, ft/s (m/s) |
| $X$ | = | position of upper mass, ft (m) |
| $\dot{X}$ | = | velocity of upper mass, ft/s (m/s) |
| $\boldsymbol{x}$ | = | generic design variable vector |
| $x_F$ | = | $X$ position of vehicle in world frame, ft (m) |
| $\tau$ | = | time constant, 1/s |

## I. Introduction

A METHODOLOGY for the effective design of reconfigurable systems is introduced. Such reconfigurable systems, as applied in this research, are defined as flexible systems, or systems that are designed to maintain a high level of performance by changing their configuration to meet multiple functional requirements or a change in operating conditions.[1] The motivation for this work comes from the area of multiobjective optimization, where there are multiple objectives or criteria that a system must simultaneously meet. Current multiobjective design practices force designers, or even disciplinary design teams, to incorporate tradeoffs into a system in an effort to resolve issues involving conflicting objectives. To obtain a functional design, it, therefore, may not be possible to achieve an optimal performance for each design objective or system component.

An approach to the effective design of reconfigurable systems is presented that first focuses on the optimization of the overall system for each known operating condition or objective. At each operating condition, the optimal values of the design variables will likely vary from that of the previous objective. Development of an effective reconfigurable system requires the understanding of this dynamic. To accomplish this, the optimal trajectory of the design variable variations is determined based on stability conditions. After the required changes in design variables are identified, it is necessary to develop a controller that will allow for effective trajectory tracking. This controller will accomplish two tasks: ensuring proper behavior of the system within a changing environment and verifying that the changes in the design variables over time are appropriate for the system considered. To complete this, the controller solution is based upon a linear quadratic regulator (LQR) approach by generalizing the process for a linear regulator problem to a linear tracking problem. Simulations of the system serve as the analytical feasibility assessment of the reconfigurable system. Further motivation and background into the development and application of reconfigurable systems are provided in the next section.

## II. Reconfigurable Systems: Motivation and Background

The primary motivation for designing reconfigurable systems arises from the need to meet multiple operating conditions, requirements, or a broad customer base. Typically, each of the various requirements that a system must meet is represented in the form of an objective function. When multiple competing objectives exist, the optimum is no longer a single design point but an entire set of nondominated design points. Commonly, this set is referred to as the Pareto set and is composed of Pareto dominant solutions.[2] A point is said to be nondominated if it is impossible to find another point strictly better in at least one objective and better or of equal performance on the other objectives. A design variable vector $\boldsymbol{x}$ is said to be Pareto optimal if and only if there does not exist another

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*Graduate Research Assistant, Department of Mechanical and Aerospace Engineering; smf7@eng.buffalo.edu. Student Member AIAA.

†Associate Professor, Department of Mechanical and Aerospace Engineering; kelewis@eng.buffalo.edu. Associate Fellow AIAA.

vector $\boldsymbol{x}'$, such that the following equations hold true:

$$f_i(\boldsymbol{x}') \le f_i(\boldsymbol{x}) \qquad \text{for} \qquad i = 1, \ldots, n \tag{1}$$

$$f_i(\boldsymbol{x}') < f_i(\boldsymbol{x}) \qquad \text{for at least one } i, \qquad 1 \le i \le n \tag{2}$$

In other words, a point is considered to be Pareto optimal if no objective can be improved without altering at least one other objective.

For a static design problem, the designer is faced with the challenge of choosing a single Pareto optimal point as the final design. However, previous research has noted that performance tradeoffs may be the result of an inferior design.[3] In addition, there is economic and political pressure to develop aircraft, as an example, that can perform multiple roles, or to be a "jack-of-all-trades."[4] This movement towards multirole vehicles can be seen with the development of unmanned aerial vehicles (UAVs). The first generations of UAVs were designed to maximize endurance and range. As the roles of these unmanned vehicles increased, so has the need for enhanced maneuverability.

To alleviate this problem, adaptability is introduced as a mode of achieving a reconfigurable system where the system design variables that can be changed, and their range of change, are identified to enhance the performance of the system. In this case, design variables are allowed to change their values physically and, therefore, the configuration and performance of the system. Through the incorporation of reconfigurable systems, it becomes possible to design a system that can satisfy the optimality conditions for multiple objective functions. Note that obtaining these optimal conditions is possible because the multiple objectives do not need to be satisfied at concurrent points in time. When it is required to satisfy multiple objectives at the same time, this approach to reconfigurable systems is not capable of entirely eliminating tradeoffs.

Examining flight in nature, NASA is using the efficient flight of birds to provide inspiration for research involving a proposed smart wing that is capable of reconfiguring, or morphing, its shape while in flight.[5] This morphing ability provides a UAV the ability to gain altitude using lift from mountains or rising thermal currents, loiter for extended periods for landing site analysis, traverse long distances with minimal loss in altitude or energy, or provide a stable platform to photograph geological structures.[6] Each of these tasks, however, requires a different airfoil shape or wing cross section for optimal performance. Currently, aircraft can extend flaps to change wing area and camber during takeoffs and landings, or use variable sweep wings for different speed regimes, using actuators and other forms of conventional technology. To advance this process, a smart material has been used to develop a high-torque rotary motor that can be configured into a wide range of formats. With a flat format, it is possible for the motor to reconfigure the curvature of the wing or fin surface of an airplane by driving changes in the camber of the aircraft wings or fins. This would essentially allow for a shape-shifting aircraft that could meet a wide range of different operating environments and mission parameters.[7]

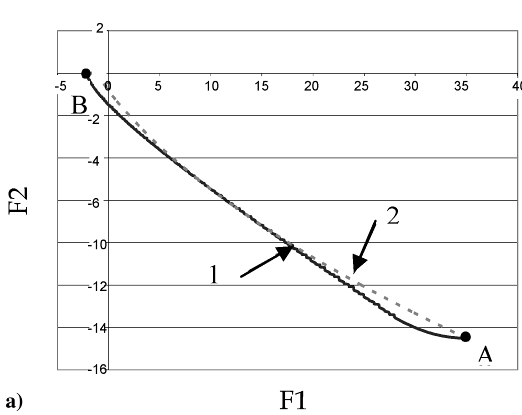
An approach to developing a morphing UAV was proposed by Rusnell[8] by designing a buckle-wing airfoil whose configuration is posed as a multilevel, multiobjective optimization problem. A buckle-wing design has the ability to change from a single wing into two wings joined at each end. Such ability provides an advantage over conventional airfoils in that the tradeoff between maneuvering and endurance/range can be partially decoupled. The computational expense of a multiobjective, multilevel buckle-wing problem is addressed by converting the problem to a single level formulation and incorporating a family of variable fidelity optimization methods.[9]

Understanding that an aircraft designed for a single mission or objective will look and behave differently from an aircraft designed for a different mission or objective, Martin and Crossley,[10] using aircraft analysis routines from McCullers,[11] studied the variation in the design variables to determine which variables would be the best candidates for morphing actuation. Roth and Crossley[12] presented the approach of treating morphing as an "independent variable" in determining which design variables should be changed and by what magnitude. This work demonstrated the ability of morphing aircraft having a lower overall takeoff weight. However, the work completed in this study was limited only to design variables associated with the aircraft wing, and the morphing was represented as occurring via an instantaneous shape change between mission segments.

The process, however, of changing from one extreme point to another is not instantaneous. Because the manner in which the system changes has direct implications on performance and the required amount of time for each modification to be completed, the path selected when changing configurations becomes inherently important. Although the most direct, and obvious, path is a straight vector between the two points, there is the possibility of violating a system constraint or losing stability of the system. However, as seen in Fig. 1, changing in accordance with the path dictated by the Pareto frontier does not automatically result in the optimal trajectory of change. (Note that both objectives $F_1$ and $F_2$ are to be minimized.)

The problem in Fig. 1 represents a purely mathematical multiobjective optimization problem. This problem calls for the minimization of objective functions $F_1$ and $F_2$ (the performance space in Fig. 1a) which are functions of the design variables $x_1$ and $x_2$ (the design space in Fig. 1b). Here, $x_1$ and $x_2$ are continuous design variables that have both upper and lower bounds. To study the transition between optimal points A and B, two different paths through the design space have been created and shown accordingly in the performance space. One path is dictated by the Pareto frontier, whereas the second path has been created with the goal of keeping the changes in the design variables as small as possible.

This example problem demonstrates the potential pitfall of following the trajectory dictated by the Pareto frontier, shown as path 1. Because both $x_1$ and $x_2$ oscillate along this path, simply moving along the frontier is unsatisfactory when designing a reconfigurable system. Such a result is not only undesirable for implementing adaptability to the system, but also for the time that it will take the system to change configurations. Also, the development of a controller for



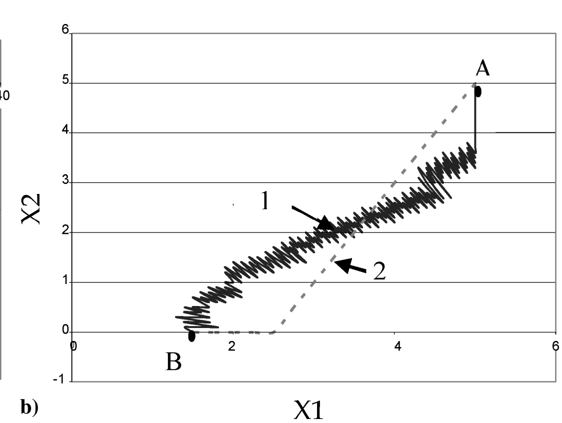




**Fig. 1 Trajectory dictated by the Pareto frontier in a) performance and b) design space.**

such a trajectory would be complex because it is difficult to reduce error, and the physical cost of such a controller could make the incorporation of adaptability infeasible. However, path 2 was created taking the system constraints defined by the optimization problem into consideration while trying to reduce the oscillations in the trajectory. What is noticed is that whereas the dramatic changes in the design variables have been eliminated, resulting in a much more controllable adaptation, there is a slight performance decrease when compared to that of the Pareto frontier. This concept has been briefly explored in work done by Prock et al.[13] focusing on minimizing the energy required by the actuators in the wing when changing airfoil shape while constraining aerodynamic performance such as lift and drag. Results of this study showed that the strain energy developed during the shape change was not only a function of initial and final positions, but also of the manner in which the shape change was completed.

Other work has focused on the process of developing missions specifically to capitalize on advantages of morphing aircraft, or methods for assessing the capability of such aircraft. Peters et al.[14] discuss an approach to generate and develop such missions complementing the "push" from the device and technology research being completed in other aspects of the aerospace industry. The missions developed could then be used in an aircraft design study to identify which parameters, features, and/or geometry should morph during flight and by what quantity. Cesnik et al.[15] describe a need to develop metrics capable of assessing morphing capabilities by systematically considering the aircraft's ability to complete the desired roles effectively based on changing its configuration. In this work, morphing was defined as "a capability to provide superior and/or new vehicle system performance by tailoring the vehicle's state to adapt to the environment and multi-variable mission roles."[15] A proposed framework was developed where the first part assesses the proposed vehicle and technology concepts with respect to the designed mission.

In the design optimization community, initial development of reconfigurable system concepts has occurred under both reconfigurable and flexible design nomenclatures. Roser and Kazmer define flexible design as having "the ability to change performance while requiring only minor time and costs to change the design parameters."[16] In this context, a flexible design methodology has been developed that improves a product design while reducing the economic cost associated with making changes to design parameters.[17,18] Incorporating reconfigurability into a structural truss optimization has resulted in reduced manufacturing costs and allows for a variety of loading conditions to be met.[19] Messac and Ismail-Yahaya provide an approach to flexible design through the incorporation of designer preferences through physical programming-based robust design optimization.[20] Meeting the needs to an ever-evolving consumer product industry is the foundation of the flexible manufacturing system, where the flexibility gained is assessed against life cycle costs and expected profits.[21,22] Application of flexible design when dealing with changing consumer demands and product platforms is studied by Suh et al. by investigating the economic impact in designing a flexible automotive floor plan.[23] Further incorporation of flexible design has been presented in the optimization of modular commercial systems that are required to respond to changing market conditions.[24]

The development of such ideas is in response to an ever-increasing complexity found in engineering design; therefore, the emergence of concepts associated with reconfigurable systems within today's industrial mindset is not a surprise. Mayflower Corporation is currently testing a variable motion engine, in which the compression ratio and the stroke of the engine may be varied through alterations of the lever-arm pivot point. When these two variables, compression ratio and capacity, are modified, it is possible to optimize the engine to meet a particular running condition. When these optimal conditions are created within the cylinder, it is estimated that fuel consumption may be reduced by nearly 40% with a corresponding 50% reduction in emissions.[25] Meanwhile, researchers at Xerox's Palo Alto Research Center are developing a new breed of robots that is flexible enough to adapt to the changing surroundings or applications.[26]

Currently, as seen from the review of related work in this background, much of today's research focuses on understanding which design variables should be modified and by how much they should change. Other work focuses on understanding when these systems would be most beneficial. In the case of UAVs, this understanding comes in the form of developing missions that are tailored for multipurpose vehicles. The work completed by Prock et al.[13] alludes to an inherent question in the design of reconfigurable systems, namely, the manner in which the design variables change from one configuration to the next. The assumption that the configuration change occurs instantaneously is a dangerous one. However, the real-time change in configuration introduces another dynamic that must be accounted for when designing a reconfigurable system. It is only through the understanding of this question that an effective reconfigurable system can actually be developed. Therefore, the optimal trajectory is determined for the design variables when changing the performance of the system to meet different conditions, the issue of performance vs ease of control needs to be taken into account. The creation of the optimal trajectory is inherently important because this information is used to develop the controller for the reconfigurable system. The development of the control scheme is discussed in the next section.

## III. Construction of a Control Law for Reconfigurable Systems

Identification of the required changes in the design variables needed to meet the multiple system requirements leads to the need of a control scheme to allow for the proper trajectory tracking of the design variables and the verification of feasibility of the reconfigurable system. A solution to the controller design problem is based on an LQR approach by generalizing the process for a linear regulator problem to a linear tracking problem.[27] This approach has been selected because LQR is generally viewed as a cornerstone of optimal control theory for dynamic systems. The development of an optimal control law seeks to satisfy the differential equation constraints and at the same time optimize some performance criterion. To accomplish this, LQR seeks an energy-based notion of weighted compromise between the closed-loop output error and a measure of the control effort required. For this type of problem, it is necessary to design a feedback control law, whose full derivation can be found in Ref. 27, that will cause the plant state to follow a given reference trajectory $\boldsymbol{r}(t)$. When the equations of motion for the system studied are linearized, the state equation is

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) \tag{3}$$

When the system representation in Eq. (3) is used, $\boldsymbol{x}(t)$ relates to the design variables, whereas $\dot{\boldsymbol{x}}(t)$ represents the time derivative of each variable. The control vector $\boldsymbol{u}(t)$ corresponds to the system inputs. In the case of a vehicle, as will be shown later, one such system input would be the vehicle's longitudinal acceleration. Finally, the $A(t)$and $B(t)$ matrices quantify the coefficients of the physical parameters of the system, taken from equations of motion. Also for this problem, the plant is desired to follow the reference trajectory $\boldsymbol{r}(t)$, such that the performance measure of the system is minimized to give the history of the final value of the tracking error in the form

$$J = \frac{1}{2}[\boldsymbol{x}(t_f) - \boldsymbol{r}(t_f)]^T S[\boldsymbol{x}(t_f) - \boldsymbol{r}(t_f)] + \frac{1}{2}\int_{t_0}^{t_f} \{[\boldsymbol{x}(t) - \boldsymbol{r}(t)]^T Q(t)[\boldsymbol{x}(t) - \boldsymbol{r}(t)] + \boldsymbol{u}^T(t)R(t)\boldsymbol{u}(t)\}\,\mathrm{d}t \tag{4}$$

where $S$ and $Q$ are real symmetric positive semidefinite matrices and $R$ is a real symmetric positive definite matrix. The $S$ matrix applies a cost to the final state deviation, $Q$ applies a cost to the state deviation over the history of time, and $R$ applies a cost to the magnitude of the input size. Although these matrices may not have a true physical meaning, other than for the development of the control law $\boldsymbol{u}^*$, the weighting scheme applied captures the designer's motivation. The $S$ matrix ensures that the design variables of the reconfigurable system will reach the correct final point of the transition. The $R$ matrix balances the amount of control that can be applied to the system, an important concern if the designer already has a certain actuator selected. The $Q$ matrix ensures that the design variables of the system follow the projected path of the transition as closely as possible. By weighting the $Q$ matrix differently, the designer can allow one, or all, design variables to deviate from the projected path to ensure stability and a proper transition between designs.

As seen in Ref. 27, the solution to the optimal LQR tracking control law is given by

$$\boldsymbol{u}^*(t) = -R^{-1}(t)B^T(t)P(t)\boldsymbol{x}^*(t) - R^{-1}(t)B^T(t)\boldsymbol{s}(t)$$
$$\equiv -G(t)\boldsymbol{x}^*(t) - \boldsymbol{v}(t) \tag{5}$$

For this control law, $G(t)$ represents the optimal feedback gain matrix and $v(t)$ is the command signal that is dependent on the system parameters and the reference signal $\boldsymbol{r}(t)$. When the $S$, $Q$, and $R$ matrices are determined, the designer must understand the tradeoffs when applying the weighting. A key to developing these tradeoffs rests in selecting the weights in proportions between the three matrices. Restricting the magnitude of the input allowed serves as a limiting control authority for the solution to the system. Having small weights in the $R(t)$ matrix may result in the need for a large control input, potentially larger than what is possible by the available equipment. The determination of the $Q(t)$ matrix can sometimes be completed by using the physical energy of the system as a guide. Managing the magnitude of each input allows for the designer essentially to dictate how great of a change each state variable is able to undergo for each discrete time step. For each of the three matrices, $S$, $Q$, and $R$, the designer selects the weighting value for the problem. When this information is used, the cost function in Eq. (4) can be determined and the control law in Eq. (5) can be developed. One of the fundamental difficulties of the LQR approach is determining the values of the weight matrices a priori. It is best to view these weight matrices as devices that require tuning to create the most efficient control laws to impose the feasibility and robustness constraints for each particular control law design.

Alternatively, it is possible to calculate the $P(t)$ and $s(t)$ matrices without the need of determining the state transition matrix, starting with

$$\dot{P}(t) = -P(t)A(t) - A^T(t)P(t) - Q(t)$$
$$+ P(t)B(t)R^{-1}(t)B^T(t)P(t) \tag{6}$$

$$\dot{\boldsymbol{s}}(t) = -[A^T(t) - P(t)B(t)R^{-1}(t)B^T(t)]\boldsymbol{s}(t) + Q(t)\boldsymbol{r}(t) \tag{7}$$

Obtaining the boundary conditions for Eqs. (6) and (7) gives

$$P(t_f) = S \tag{8}$$

$$\boldsymbol{s}(t_f) = -Sr(t_f) \tag{9}$$

To obtain the optimal control law for the system, integrate the differential equations of Eqs. (6) and (7) backward in time starting with the terminal boundary conditions of Eqs. (8) and (9). Storing the results of this integration as $P(t)$ and $s(t)$ will allow for the determination of $G(t)$ and $v(t)$ in Eq. (5).

The development of the optimal control law provides the designer with information regarding the feasibility and practicality of implementing the reconfigurable system design for the reference trajectory created. For situations where the needed control is required to be large to maintain a minimum tracking error, it may be necessary to reduce the requirements of the reconfigurable system. This slight performance loss will be compensated by the development of a practical reconfigurable system design. To demonstrate the design and application of a reconfigurable system, a case study is introduced regarding the design of a Formula One race car traversing a straightaway.

## IV. Case Study: Design of a Formula One Race Car

In the design of a race car, the difference between winning a race and not winning comes down to the ability of a driver to get the most out of the vehicle. The core vehicle design is aimed at an optimal compromise that allows the driver to turn fast lap times repeatedly at a particular racetrack. Simulations are used not only to tune the vehicle on the weekend of the race, but also in the design phase where parameters that are not adjustable must be set and optimized. The basic configuration of the car, such as the center of gravity, suspension systems, and roll stiffness, remain constant. A particular vehicle design will be optimal at a certain speed and radius of the curvature. Formula One racetracks do not have constant radius corners and do not consist of only a few turns; however, the design team must choose a single vehicle configuration on race day.

Now, consider a reconfigurable racecar design that is able to optimize its performance as a function of the current track conditions, ignoring current racing restrictions. Whether on a straightaway, a large turn, or a hairpin turn, the car could adjust variables such as the center of gravity, roll stiffness, and aerodynamic downforce (via wings and airfoils). The objective of this problem is to design a reconfigurable race car that has the ability to adapt itself to minimize the time along a portion of a racetrack consisting of two corners of different radii and a connecting straightaway. To describe the motion along such a course, the vehicle enters the straightaway at the maximum steady-state cornering velocity from the previous corner. It then accelerates as hard as possible until, at the last possible instant, it brakes as hard as possible so that at the end of the straightaway the vehicle's velocity is equal to the maximum steady-state cornering velocity of the upcoming corner. The simulation in this study is completed by evaluating each portion of the track separately. Although this simulation does not capture every aspect of the vehicle's dynamics, the results provide enough information to capture the design performance and configuration for each portion of the track. Figure 2 shows an example of the racetrack segment used in this case study.

Figure 3 shows the four steps involved in designing a reconfigurable system. The first step, as seen in the next subsection, involves any necessary model development. Next, the multiobjective optimization problem is stated and solved. Solving this problem will give the designer the information about the Pareto frontier needed to identify the optimal extreme points. This information also allows for the creation of the different paths of change for the design variables to be determined. The change in design variables allows for a unique control law to be developed for each reconfigurable case. Finally, the results of each path and the overall performance of the reconfigurable system can be analyzed to yield the best reconfigurable system.

### A. Model Development

The amount of detail that can be modeled in a computer-based simulation of an automobile is almost limitless. Although many of the details are unnecessary in the preliminary design stage, it is necessary to model the basics of the vehicle design correctly. A brief explanation of the race car model is presented in Ref. 28. The design variables chosen for this model represent three potential disciplines working on the vehicle. The variables corresponding to the longitudinal center-of-gravity location $a/\ell$, roll stiffness distribution $K_F/(K_F + K_R)$, and the aerodynamic downforce distribution $C_{\ell F}/(C_{\ell F} + C_{\ell R})$ are also the primary parameters that affect a vehicle's performance.[29] Figure 4 is a basic representation of the vehicle model under consideration.

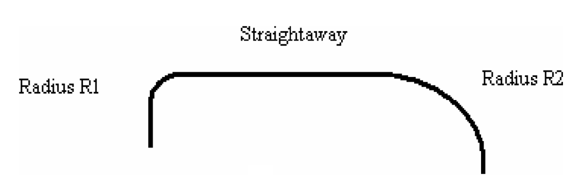



**Fig. 2 Example of case study racetrack.**

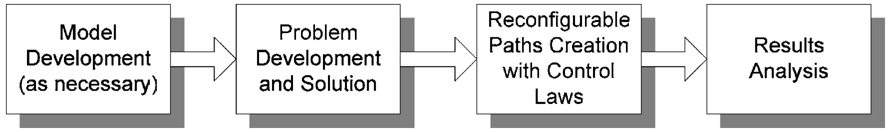



**Fig. 3 Flowchart for design of reconfigurable systems.**

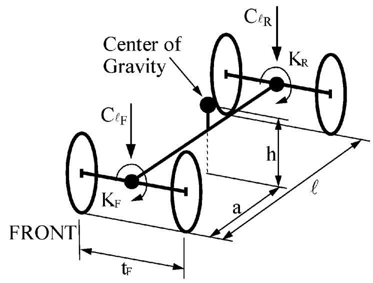



**Fig. 4 Vehicle model simplification.**

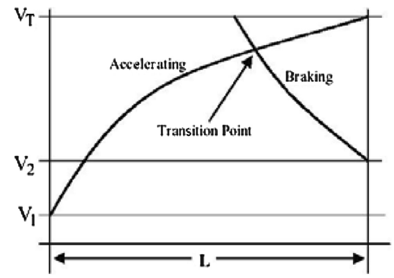



**Fig. 5 Components of straightaway performance.**

The fore/aft distance of the vehicle's center of gravity behind the front axle, divided by the vehicle's wheelbase $\ell$, represents the weight distribution. Aerodynamic downforce distribution is the division of individual aerodynamic downforce acting at the front and rear axles. This force is created by the overall vehicle shape and the inverted airfoils. Roll stiffness distribution signifies the amount of resistance to vehicle roll the front axle provides relative to the total resistance provided by the front and rear tires. Each of these design variables is normalized between 0 and 1, as follows:

Normalized longitudinal center of gravity location

$$a' = a/\ell = a/(a + b) \tag{10}$$

Normalized roll stiffness distributions

$$K' = K_F/(K_F + K_R) \tag{11}$$

Normalized aerodynamic downforce distribution

$$C' = C_{\ell F}/(C_{\ell F} + C_{\ell R}) \tag{12}$$

The detail present in this model is sufficient for the analysis of the basic design concept for the optimization of the vehicle. Analysis of the vehicle design when in a turn of given radius is solely done in the condition of steady-state cornering. When iterative solution techniques are used, the constant velocity at peak cornering, represented by maximum lateral acceleration, on a skidpad is found. The performance of this vehicle model on variable radius skidpads has been extensively studied in Ref. 30, and a more detailed look at the construction of the vehicle may be found in Ref. 31. The analysis behind the simulation of a race car on a straightaway between corners of two different radii is developed in Ref. 31. The performance on the straightaway is composed of an accelerating and braking phase, as seen in Fig. 5, to match the restrictions placed upon the system by the radius of the two corners. Therefore, one of the main obstacles of this analysis is the uncertainty regarding the location of the transition point between the acceleration and braking phases because it is a function of the curve radii, the length of the straightaway, and the parameters of the vehicle. In Fig. 5, $V_1$ corresponds to the maximum steady-state velocity of the first radius, $V_2$ the maximum steady-state velocity of the second radius, and $V_T$ the terminal velocity of the vehicle along the straightaway.

The transition point is solved by finding the maximum speeds $V_1$ and $V_2$ the vehicle may obtain on the two corners defined in Fig. 2. Start at speed $V_1$, then the acceleration phase of the vehicle is modeled to find the terminal velocity $V_T$. From this terminal speed, the braking profile is determined, slowing the vehicle to the maximum steady-state velocity of the second radius. The intersection of the acceleration and braking profile is the solution for the transition point, as shown in Fig. 5, and must be solved whenever the radii of the corners or length of the straightaway changes. The unique solution of this profile serves as the reference trajectory for each reconfigurable system studied.

In the development of this model, the increased costs due to changes in the design variables have not been accounted for. For example, these costs should take into account any additional engineering and development time required, foreseeable additions to the manufacturing process, and any material costs that will be incurred. Also, it is possible to account for the increased costs due to changing the existing initial design. For instance, placing an extra motor onto the vehicle may result in the need for a larger structural support. This extra need will change the weight of the vehicle and change the overall performance. For this work, the focus is on the successful determination of a control law to design the most effective reconfigurable system. The concept of handing the costs and associated complexities into reconfigurable systems may be found in Ref. 32.

### B. Problem Development and Solution

The multiobjective problem developed by this analysis is defined as the minimization of the time it takes to traverse each section of the proposed racetrack, subject to the constraints placed on the initial and final values of the vehicle's velocity as determined by the maximum attainable steady-state cornering speed for a given configuration. This optimization problem can be written in standard form as follows:

Minimize

$$F_1 = \text{time to traverse radius } R_1\text{, s} \tag{13}$$

$$F_2 = \text{time to traverse a straightaway of length } L\text{, s} \tag{14}$$

$$F_3 = \text{time to traverse radius } R_2\text{, s} \tag{15}$$

subject to

1) the initial velocity of the straightaway, $V_1$, as dictated by the maximum steady-state cornering velocity of turn 1 (16)

2) the final velocity of the straightaway, $V_2$, as dictated by the maximum steady-state cornering velocity of turn 2 (17)

$$0.45 \le a' \le 0.65 \qquad \text{(normalized center of gravity position)} \tag{18}$$

$$0.4 \le K' \le 0.7 \qquad \text{(normalized roll stiffness)} \tag{19}$$

$$0.25 \le C' \le 0.45 \qquad \text{(normalized lift coefficient)} \tag{20}$$

The normalized ranges of the three design variables initially had an upper and lower bound of 1 and 0, respectively, whereas the ranges used in Eqs. (8–20) are selected to reflect realistic results. For this problem, the first turn in the proposed racetrack has a radius

**Table 1 Identification of Pareto extreme points for the vehicle optimization problem**

| Endpoint | $a'$ | $K'$ | $C'$ | $F_1$, s | $F_2$, s | $F_3$, s |
|---|---|---|---|---|---|---|
| A: 42.67-m (140-ft) radius | 0.65 | 0.63 | 0.32 | 11.156 | 7.746 | 14.627 |
| B: 381-m (1250-ft) straightaway | 0.45 | 0.40 | 0.45 | 11.428 | 6.514 | 15.628 |
| C: 121.92-m (400-ft) radius | 0.65 | 0.48 | 0.31 | 11.175 | 7.801 | 14.531 |

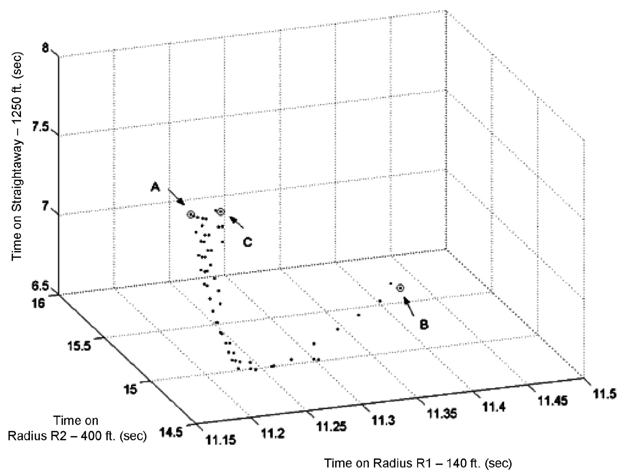



**Fig. 6 Pareto frontier for race car optimization problem in performance space.**

of 42.67 m (140 ft). This radius, from which $V_1$ is found, merges into a 381-m (1250-ft) straightaway. The vehicle then traverses the straightaway, turning into the second corner with a radius of 121.92 m (400 ft). Figure 6 shows the three-objective performance space view of the Pareto frontier. From the results of the Pareto frontier, the optimal configuration for each objective can also be identified. The values of these points are listed in Table 1. This multiobjective problem was solved using a grid search over the established bounds of the design variables, given the ease of integrating the iterative approach required to analyze each evaluation. Figure 6 is a three-dimensional plot of the Pareto points resulting from the grid search. Each axis of this plot represents the time required to complete one lap on each portion of the track. In the cases of objectives $F_1$ and $F_3$ , the values relate to the time required to complete one full revolution around the skidpad. For objective $F_2$, the value is the time to traverse the length of the straightaway portion of the track. The optimal points for each objective, A, B, and C in Fig. 6, have the coordinates shown in Table 1.

### C. Development of Control Law

To make the system reconfigurable for the different operating objectives, two scenarios must be investigated. The first scenario involves changing the vehicle from the optimal configuration of the first radius (point A) to the optimal configuration of the straightaway (point B). The second scenario involves changing the vehicle from the optimal configuration of the straightaway (point B) to that of the second, upcoming corner (point C). As discussed earlier, a design variable adapts itself in discrete increments to attain the values of the Pareto extreme points. To obtain an optimal configuration during the period that the design variables are changing, the path of change would be that of the Pareto frontier. By the movement along the Pareto frontier, the system retains a degree of optimality, in that it never takes on the configuration of a dominated design. However, the path dictated by the Pareto frontier may not be the most efficient or effective solution to incorporating flexibility, and may therefore, be potentially eliminated as a desired solution, as was discussed earlier in relation to Fig. 1.

When alternative paths in the design space are determined and then mapped accordingly to the performance space, it is possible to develop a more effective and efficient reconfigurable system. The most obvious deviation from the Pareto frontier path is a linear vector between the two Pareto extreme points in the design space. However, taking such a route could result in the violation of system constraints, thereby eliminating this path as a possible solution. Therefore, a path must be constructed in the design space that takes into account both the efficiency of the adaptation and the constraints of the overall system. To determine the characteristics of these alternative paths, the following suboptimization problem is presented for the design of reconfigurable systems:

Minimize

$$F_j(\boldsymbol{x}) + \sum_{i=1}^{n} \left(x_i^q - x_i^B\right)^2 \tag{21}$$

subject to

$$\left(x_i^q - \Delta x_i\right) \leq x_i^q \leq \left(x_i^q + \Delta x_i\right) \qquad \text{for} \qquad i = 1, \ldots, n \tag{22}$$

$$x_i^L \leq x_i \leq x_i^U \qquad \text{for} \qquad i = 1, \ldots, n \tag{23}$$

$$g_k \leq 0 \qquad \text{for} \qquad k = 1, \ldots, m \tag{24}$$

This suboptimization problem essentially places a hyperbox around the current design point in the design space and searches for the location within this box that reduces the objective function selected. The size of the hyperbox will change the solution of the suboptimization problem. A larger hyperbox will not only take less time to change configurations, but may also require a device with greater power. Also, it has been seen that different sized hyperboxes will follow different paths through the design space to complete the configuration change. Equation (21) comprises two parts, with the first part being the objective that is to be minimized. The second part of Eq. (21) calculates the distance from the current design vector to the Pareto extreme point that the system is attempting to reach. The combination of these terms serve to ensure that the alternative path begins and terminates at the specified Pareto extreme point while ensuring that the method does not converge to a local minimum and that the total change in the design variables is minimized. Equation (22) establishes the boundary of the hyperbox, essentially pseudoside constraints for each design variable, for all iterations of the optimization subproblem. Equations (23) and (24) serve to incorporate the original constraints placed on both the design variables and the system being studied.

Graphically, this procedure can be shown as follows. In Fig. 7, two points are selected (A and B) that map to Pareto extreme points. For this two-objective, two-design variable problem, the path dictated by the Pareto frontier is represented by a dashed line and point A is selected as the starting point. When a hyperbox is placed around the starting point, the subproblem objective function is minimized. The solution to this optimization problem creates a new point in the design space, as seen in Fig. 8. Mapping this point over to the

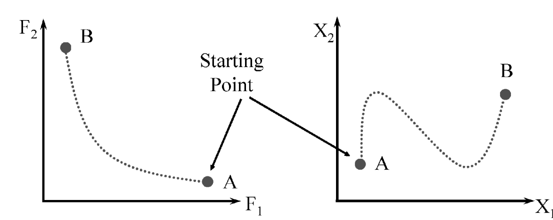



**Fig. 7 Selection of starting point for optimization subproblem.**

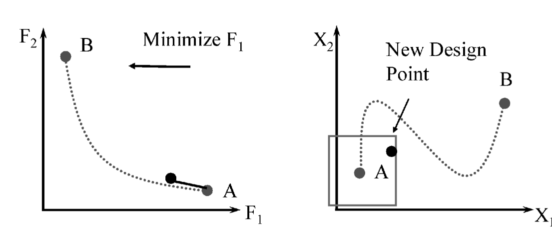



**Fig. 8 First iteration of optimization subproblem.**

performance space provides the first increment in developing the performance frontier for the newly created path. Then a new hyperbox is constructed around the current design point, and the process is repeated until reaching point B in the design space. The next iteration of the subproblem is shown in Fig. 9. Note that it is assumed that the objective functions are dependent on all of the design variables, in that a design variable cannot be changed independently to influence one objective without affecting the other objectives. Otherwise, a series of one-dimensional design variable changes could be easily determined.

After the values of the selected objective function are minimized for all iterations within the fabricated hyperbox, it is possible to develop an efficient path of change that closely approximates the Pareto frontier. These five cases are identified in Table 2, and the plots for each path are shown in Fig. 10, corresponding to the design space for the two different configuration changes. Case 1 comes from the Pareto frontier, and case 2 comes from the linear vector connecting the two Pareto extreme points. Cases 3–5 were created using the optimization subproblem defined by Eqs. (21–24) with different boundaries used to create the hyperbox, resulting in different paths through the design space as seen in Fig. 10. Figure 11 shows how the change in design variables for each reconfigurable system affects performance space results. For both transitions, the suboptimization cases 3–5 provide a better performance than the linear path (case 2) by giving a better approximation of the Pareto frontier path.

With the development of the possible paths of change for the system, it is necessary to ensure that the proposed paths are both feasible and practical. The states for the system are defined as the position and velocity of the vehicle and the value of the three design variables. When the state-space model is developed, the changes in configuration occur only when the vehicle is on the straightaway and there is no modeling of the transition from steady-state cornering to the straightaway, or vice versa. This allows for the elimination of lateral forces from the state-space model, simplifying the model while not deterring from the effectiveness of the analysis.

To determine the equation of motion for the vehicle in terms of the state variables, it is necessary to first study the forces acting on the system. Detailed analysis and explanation of these equations of motion can be found in Refs. 28–31. To incorporate adaptability, the center of gravity is varied by breaking the vehicle up into a chassis mass and an upper-body mass. In doing so, it becomes possible to displace the upper-body mass to change the center of gravity. To accomplish this, the state variable $X$ is introduced, representing the location of the upper mass with respect to the front of vehicle. For a standard race car, a nominal value of $a'$ is 0.59524. For this study, the chassis will represent 75% of the total mass of the vehicle. The upper body (the moveable portion of the race car) consists of the remaining 25% of the vehicle mass. These mass breakdowns were arbitrarily selected for this study and could be treated as a design variable in future work. When these percentages are used, $a'$ is expressed in terms of $X$:

$$\begin{aligned} a' &= 0.75(\text{c.g. of chassis}) + 0.25(\text{c.g. of upper mass}) \\ &= \tfrac{1}{4}[(x/\ell) + 1.78572] \end{aligned} \tag{25}$$

The new listing of states used in the state-space model is shown in Table 3.

Modeling of the equations of motion for the roll stiffness distribution and the aerodynamic downforce distribution is completed by using a first-order differential equation:

$$\dot{C}'(t) + (1/\tau_C)C'(t) = F_C(t) \tag{26}$$

$$\dot{K}'(t) + (1/\tau_k)K'(t) = F_K(t) \tag{27}$$

However, the resulting acceleration equation is nonlinear in nature and needs to be linearized with respect to each state variable and

**Table 2 Identification of the different paths studied to achieve flexibility**

| Path number | Path identification | Hyperbox side constraints $a'$ | $K'$ | $C'$ |
|---|---|---|---|---|
| Case 1 | Pareto path | ±0.05 | ±0.05 | ±0.05 |
| Case 2 | Linear path | ±0.025 | ±0.07 | ±0.05 |
| Case 3 | Suboptimization, case 1 | ±0.05 | ±0.05 | ±0.05 |
| Case 4 | Suboptimization, case 2 | ±0.025 | ±0.07 | ±0.05 |
| Case 5 | Suboptimization, case 3 | ±0.025 | ±0.1 | ±0.075 |

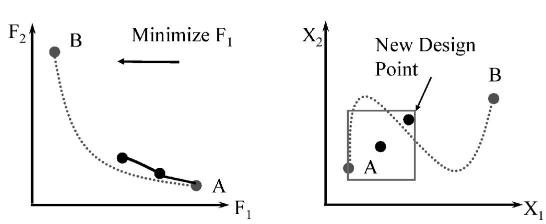



**Fig. 9 Second iteration of optimization subproblem.**

**Table 3 State listing for state-space model**

| States | Definition |
|---|---|
| $x_F$ | $x$ Position of vehicle in world frame |
| $u$ | Longitudinal velocity |
| $K'$ | Normalized roll stiffness distribution |
| $C'$ | Normalized aerodynamic downforce distribution |
| $X$ | Position of upper mass |
| $\dot{X}$ | Velocity of upper mass |

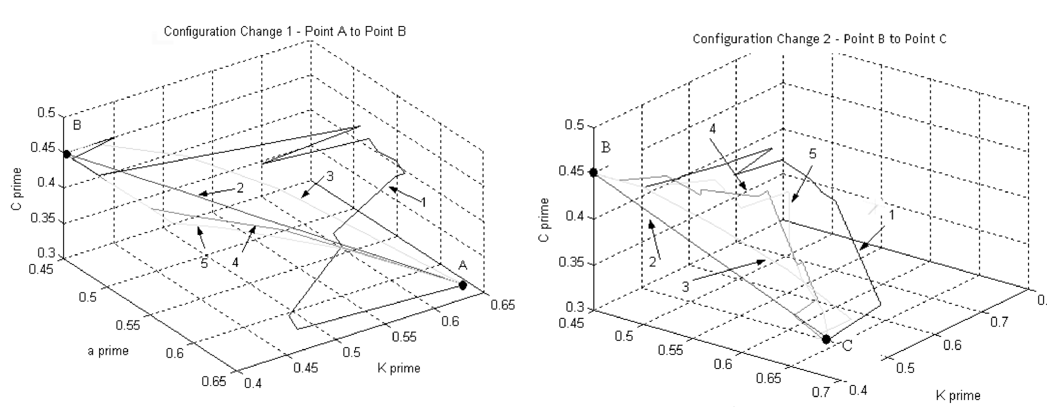



**Fig. 10 Design space of proposed reconfigurable systems for both configuration changes.**

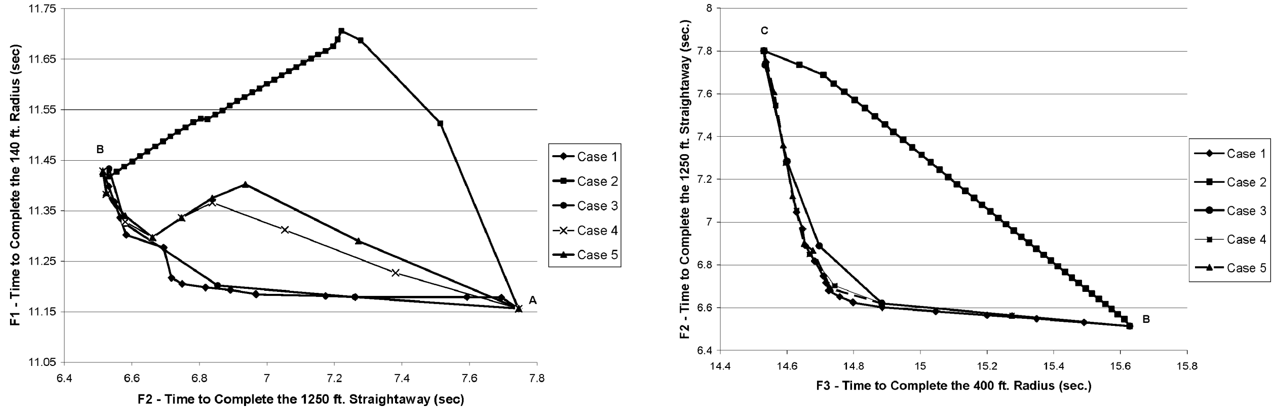



**Fig. 11 Performance space of proposed reconfigurable systems for both configuration changes.**

input, such that it can be used in the state-space form, as

$$\begin{Bmatrix} \dot{x}_F \\ \dot{u} \\ C' \\ K' \\ \dot{X} \\ \ddot{X} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \dfrac{\partial \dot{u}}{\partial x_F} & \dfrac{\partial \dot{u}}{\partial u} & \dfrac{\partial \dot{u}}{\partial C'} & \dfrac{\partial \dot{u}}{\partial K'} & \dfrac{\partial \dot{u}}{\partial X} & \dfrac{\partial \dot{u}}{\partial \dot{X}} \\ 0 & 0 & -\dfrac{1}{\tau_C} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{1}{\tau_K} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\dfrac{K_U}{m_U} & -\dfrac{c_U}{m_U} \end{bmatrix} \begin{Bmatrix} x_F \\ u \\ C' \\ K' \\ X \\ \dot{X} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \dfrac{\partial \dot{u}}{\partial A_X} & \dfrac{\partial \dot{u}}{\partial F_C} & \dfrac{\partial \dot{u}}{\partial F_K} & \dfrac{\partial \dot{u}}{\partial F_X} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} A_X \\ F_C \\ F_K \\ F_X \end{Bmatrix} \tag{28}$$

where the values of the partial derivatives are evaluated at each time step from the values of the reference trajectory to update the $A$ and $B$ state-space matrices. As already stated, the profile in Fig. 5 contains the information of the first two states of the optimal configuration, position and velocity. To account for the remaining states, the path information from Fig. 10 is substituted into the reference trajectory. When this system is evaluated, it is of extreme importance that the vehicle is going no faster than $V_2$ when entering the second turn because that is the maximum steady-state speed at which control of the vehicle can be maintained. For this study, the initial velocity was found to be 24.03 m/s (78.85 ft/s) and the final velocity was found to be 52.72 m/s (172.96 ft/s). With this information known, the cost equation used in the linear tracking problem can be constructed:

$$J = \frac{1}{2}[x(t_f) - r(t_f)]^T \times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} [x(t_f) - r(t_f)] + \frac{1}{2}\int_{t_0}^{t_f} \left\{ [x(t) - r(t)]^T \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times [x(t) - r(t)] + u^T(t) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} u(t) \right\} \mathrm{d}t \tag{29}$$

When the cost function is constructed, the final error matrix is weighted in such a way that emphasis is placed on the three design variables. Similarly, the state deviation matrix is weighted to place importance on the deviations of the design variables as they follow the reference trajectory. The input size is not of major importance to this study and, therefore, is equally weighted for all states. With the cost function defined, the optimal control law for the system is developed, allowing for the simulation of the system to determine the true path of change. When the first transition (A–B) of case 5 is used as an example, the true path of the design variables are studied against the supplied reference trajectory. The second transition (B–C) also demonstrates the same behavior, and hence, the same conclusions can be drawn. Figure 12 shows the movement of $C'$ over the transition period of 2 s. It can be seen that there is a slight deviation in the final value. Examination of Fig. 12b shows that $K'$ is almost perfectly tracked by the system, ending at the exact final value and following all intermediate transitions.

When the information from the state variable $X$ corresponding to the position of the upper mass is converted back into the design variable $a'$, it is possible to plot the trajectory of the center of gravity under the influence of the optimal control law. Because the upper mass was modeled as a spring–mass–damper system, it was predicted that the results of the true path would not correspond perfectly to the reference trajectory. This was expected as the reference trajectory was not created with the modeled system in mind, but instead by the result of the suboptimization algorithm.

In Fig. 13, the response of $a'$ to the reference trajectory is shown. As is expected, the second-order system is not able to follow the nearly linear path of the reference trajectory. However, this variable does reach and remain constant at its final value of 0.45 after the 2-s change in configuration. These simulations are completed for the other paths of change, allowing for performance of each reconfigurable system to be determined.

### D. Results Analysis and Race Simulation

For this problem, the results of the five reconfigurable systems, originally presented in Table 2, are compared to the results of three static race cars. Each static vehicle was designed to be optimal for one of the three objectives studied in this problem. For example, car A is designed such that its configuration is optimal for the first radius. Cars D–H are designed to be reconfigurable vehicles. For these race cars, the incorporation of adaptability allows for the attainment of the optimal configuration for all objectives. As seen in Table 4, the reconfigurable vehicles each complete the two different radii in the minimum possible time.

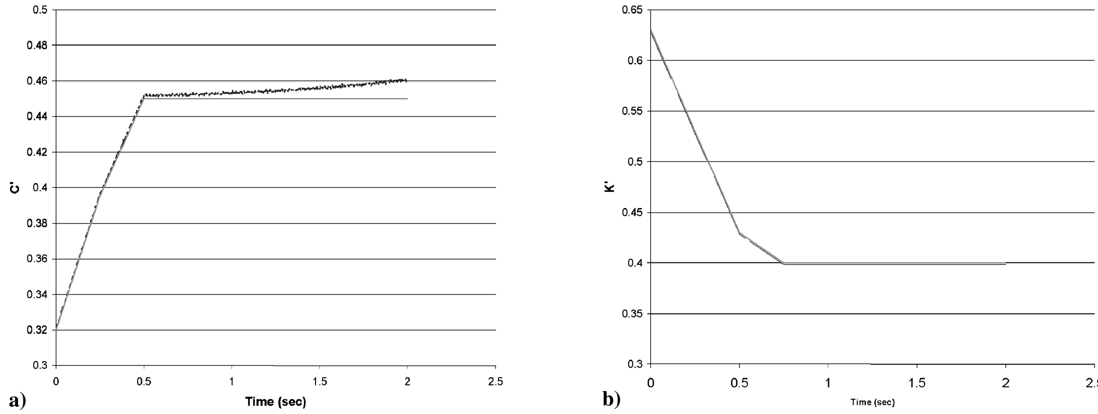



**Fig. 12 Simulation of $C'$ and $K'$ compared to reference trajectory for first transition (A–B): - - - -, state simulation and · · · ·, reference trajectory.**



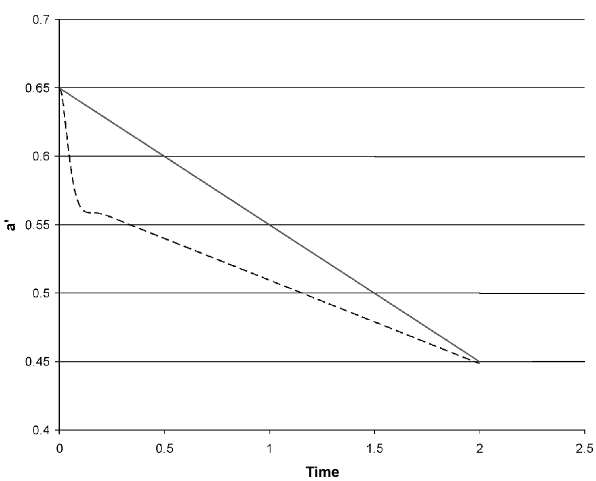



**Fig. 13 Simulation of $a'$ compared to reference trajectory for first transition (A–B): - - - -, state simulation and · · · ·, reference trajectory.**

**Table 4 Simulation results for system objectives for field of race cars**

| Car | Configuration | $F_1$, s | $F_2$, s | $F_3$, s |
|---|---|---|---|---|
| A | Optimal configuration for radius 1 | 11.156 | 7.746 | 14.627 |
| B | Optimal configuration for radius 2 | 11.175 | 7.801 | 14.531 |
| C | Optimal configuration for straightaway | 11.428 | 6.514 | 15.628 |
| D | Reconfigurable system, Pareto frontier | 11.156 | 6.753 | 14.531 |
| E | Reconfigurable system, linear path | 11.156 | 6.678 | 14.531 |
| F | Reconfigurable system, suboptimization case 1 | 11.156 | 6.546 | 14.531 |
| G | Reconfigurable system, suboptimization case 2 | 11.156 | 6.642 | 14.531 |
| H | Reconfigurable system, suboptimization case 3 | 11.156 | 7.503 | 14.531 |

The optimal control law solutions from the preceding section determine the paths of change for the design variables, allowing for the simulation of the vehicles on the straightaway. Because the vehicle is modeled at steady-state cornering in this model, the reconfigurable systems only change configuration when on the straightaway. Based on this, different paths for the design variables lead to different times required to traverse the straightaway. The simulated times for the three objectives in this problem are shown in Table 4. The objective function values shown represent the time required to traverse each skidpad or straightaway once. The path of change for car D is dictated by the information of the Pareto frontier. The path of change for car E comes from the linear vector connecting the two optimal configurations under consideration. The solutions for cars F–H are dictated by the results of the suboptimization problem. For these cases, the solution is a function of the boundaries placed on the hyperbox.

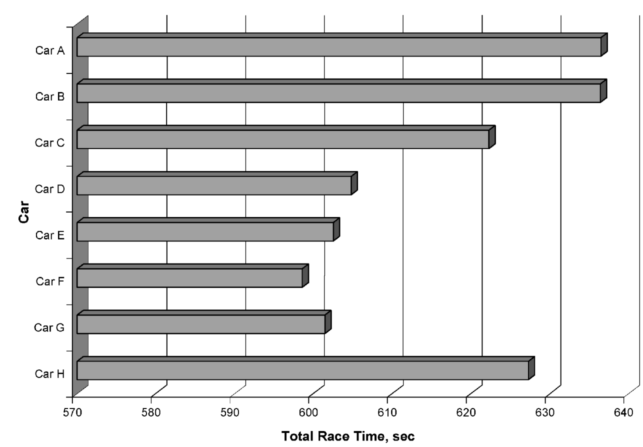



**Fig. 14 Results for field of race cars for simulated race.**

Simulation of a race was completed using a racetrack represented by 10 times around a skidpad of 42.67-m (140-ft) radius, 20 times around a skidpad of 121.92-m (400-ft) radius, and 30 times down a straightaway of length 381 m (1250 ft). Whereas the exact number of executions of each segment of the racetrack is arbitrary, the general race profile has been developed with help from Milliken Research Associates, an automotive company with vast experience in Formula One race car consulting.

The results of this race are shown in Fig. 14. From Fig. 14, it can be seen that all but one of the reconfigurable race cars finish faster than the three static configurations tested. Also, cars F and G provide the best performance of all of the vehicles analyzed in this study. The slower performance of cars D and E demonstrate that the paths dictated by the Pareto frontier and the linear vector through the design space do not always provide the best performance. This can be validated by examining Fig. 10, where the overall design variable changes for the Pareto frontier path are much greater than the other reconfigurable systems. Figure 11 shows that the path dictated by the linear path is the farthest from the Pareto frontier in the performance space for both transitions. However, the reconfigurable systems usually perform noticeably better than their static counterparts. An occurrence of worse performance by a reconfigurable system can be seen when examining the design variables changes for car H. Given the changes required, car H is able to obtain optimal performance for only a small portion of time on the straightaway. This duration is not long enough to produce a significant performance change, causing the vehicle to perform worse than the static car C. Although the reconfigurable car H outperforms car C on the two radii, the faster performance of car C outweighs the advantage of being reconfigurable.

## V. Conclusions

This paper presents a procedure for designing reconfigurable systems using a multiobjective optimization problem, eliminating the need for making tradeoffs between conflicting objectives. Incorporating adaptability allows for the configuration of a system to change, thereby altering the system's performance. In this work, three different ways for determining the manner in which the design variables can be changed are introduced. The first method follows the path dictated by the Pareto frontier solution, and the second method uses the information from the linear vector in the design space that connects two of the optimal configurations. However, the linear path may violate system constraints that compose the problem. The third method applies an iterative optimization subproblem that incorporates the original system constraints. For this subproblem, a hyperbox is created in the design space, whose size is selected by the designer. The series of solutions to these subproblems become the paths of change for the system. Next, the stability of the system when changing configurations is analyzed through the application of a linear-tracking problem. The application of an established control law designed for tracking problems, as described in this work, is used to help the design teams ensure that the proposed reconfigurable systems are both stable and practical.

Applying this approach to the design of a Formula One race car demonstrates the potential benefits of designing reconfigurable systems. For a proposed racetrack, consisting of two corners and a straightaway, the multiobjective problem was solved to identify the optimal configuration for each of the three objectives. Developing the different potential paths of change for the design variables was completed using the three different approaches. The equations of motion of the vehicle traveling along the straightaway were then used in the stability analysis of the systems. Because the vehicle's operation in the corners was modeled under steady-state cornering, the changes in configuration of the vehicle occur only when the vehicle is traveling along the straightaway. When compared to their static counterparts, almost all of the reconfigurable vehicles were successful in attaining an increased overall performance. When the different vehicles were evaluated, it was seen that the vehicle designs developed under the optimization subproblem outperformed the vehicle designed by either the Pareto frontier or the linear vector. The significantly increased performance of the reconfigurable systems over their static counterparts provides support that reconfigurable system design could become a major area of interest for further multiobjective problems.

The results of this work also provide interesting insight into the design of reconfigurable aircraft. Although there are many aspects of basic reconfigurability in today's aircraft, the literature review in Sec. II demonstrates the need for further complexity. Modeling of a system allows for the development of a control law for reconfigurable systems, resulting in the true path of change for the design variables. Understanding how this true path may affect aircraft performance is of great importance because loss of stability or effectiveness may result in disastrous results. With this approach, stability during the transition phase of reconfiguration can be studied and ensured directly.

A source of future work involves examining cases where it may be impossible, or too great of an engineering requirement, to make certain design variables adaptable. In these cases, an alternative approach is needed. When designed for robustness, a given design variable will remain constant at a value that would be optimal for a wide range of operating conditions or objectives. Further study of the relationships and interactions when applying adaptability and robustness to a system is essential for the future development of more complex reconfigurable systems. There may also be cases where it may be impossible, or implausible, to achieve the optimal performance. Finally, the work presented in this paper is theoretical in nature. Through computer simulations, the practicality and benefits of reconfigurable systems can be determined, but the need exists to construct an actual reconfigurable system for testing.

## Acknowledgments

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A. Messac
*Associate Editor*