Exploring Product Solution Differences Due to Choice Model Selection in the Presence of Noncompensatory Decisions With Conjunctive Screening Rules

Jaekwan Shin
Department of Mechanical and Aerospace Engineering,
North Carolina State University,
911 Oval Drive,
Raleigh, NC 27695
e-mail: jshin5@ncsu.edu

Scott Ferguson[1]
Associate Professor
Department of Mechanical and Aerospace Engineering,
North Carolina State University,
911 Oval Drive,
Raleigh, NC 27695
e-mail: scott_ferguson@ncsu.edu

Research in market-based product design has often used compensatory preference models that assume an additive part-worth rule. These additive models have a simple, usable form and their parameters can be estimated using existing software packages. However, marketing research literature has demonstrated that consumers sometimes use noncompensatory-derived heuristics to simplify their choice decisions. This paper explores the quality of optimal solution obtained to a product line design search when using a compensatory model in the presence of noncompensatory choices and a noncompensatory model with conjunctive screening rules. Motivation for this work comes from the challenges posed by Bayesian-based noncompensatory models: the need for screening rule assumptions, probabilistic representations of noncompensatory choices, and discontinuous choice probability functions. This paper demonstrates how respondents making noncompensatory choices with conjunctive rules can lead to compensatory model estimations with distinct respondent segmentation and relative, large absolute part-worth values. Results from a product design problem suggest that using a compensatory model can provide benefits of smaller design errors and reduced computational costs. Product design optimization problems using real choice data confirm that the compensatory model and the noncompensatory model with conjunctive rules provide comparable solutions that have similar likelihoods of not being screened out when using a consideration set verifier. While many different noncompensatory heuristic rules exist, the presented study is limited to conjunctive screening rules. [DOI: 10.1115/1.4035051]

1 Introduction

Companies continually strive to better understand customer preferences for product performance and feature inclusion so that they can be successful in a globally competitive market. Market-driven product design research has explored the use of discrete choice analysis as one tool for estimating customer preferences by implementing variations of the generalized linear model [1–4]. Many discrete choice model forms assume that consumers make compensatory choices based on an additive utility rule; that is, high levels on some features can compensate for low levels on other features.

However, market research papers have demonstrated that noncompensatory choice models often improve model realism and accuracy in predicting consumers' choices [5–8]. Imagine a consumer, who does not want a manual transmission, shopping for a new car. The consumer first uses a heuristic rule to narrow their choice to a set of cars equipped with an automatic transmission. The remaining cars are then compared using an additive utility rule. This choice behavior is called a consider-then-choose process.

Since the early 2000s, there has been increased development in modeling noncompensatory choices using computationally expensive methods like Bayesian inference and machine learning techniques. However, the effectiveness of using noncompensatory models in a product design search has not been extensively explored. Nonlinear programming relaxations for market-system design optimization problems were proposed in Ref. [9] to deal with the discontinuous likelihood functions of a consider-then-choose model. This work was extended in Ref. [10] to investigate how noncompensatory choice behavior impacted profit when making design decisions. These studies, however, focused on evaluating predictive power and design error at the population level without exploring individual-level part-worth estimates.

The objective of this paper is to compare the optimal product line solutions when using Bayesian-based noncompensatory models with conjunctive screening rules and a compensatory hierarchical Bayes mixed logit (HB-ML) model. This study is motivated by the challenges of making screening assumptions and inferring the screening rules used by a respondent population. Even when these screening rules can be correctly inferred, estimating a two-stage model can be challenging, leading to errors that can lead to suboptimal design decisions. Further, the probabilistic representations of noncompensatory choice and the discontinuous choice probability functions that often accompany noncompensatory models make optimization more challenging.

The suitability of using existing noncompensatory models is discussed in Sec. 2 by reviewing the existing models. In particular, Sec. 2.2 addresses the limitations of an existing noncompensatory model. Section 3 describes how to explore the performance of using a compensatory model in the presence of noncompensatory choices. This concept is examined in Sec. 4 by analyzing synthetic data. Section 5 deals with real choice data to explore

[1]Corresponding author.

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differences in a product optimization using both the compensatory and noncompensatory models.

2 Background

The necessary background knowledge about discrete choice models is introduced to aid in the explanation of this study. Section 2.1 briefly reviews the fundamental concepts of compensatory models capable of estimating individual-level part-worths—latent class multinomial logit (LC-MNL) and hierarchical Bayes mixed logit (HB-ML). In Sec. 2.2, various heuristics of noncompensatory choices and their modeling methods are reviewed, mainly focusing on the HB multinomial probit model with conjunctive screening rule.

2.1 Compensatory Choice Models. The assumption behind compensatory models is that consumers weigh and compare all available attributes across all products before making a selection. Discrete choice analysis is used to model product demand by capturing a customer's choice behavior [11]. The choice utility that person n obtains from alternative i can be expressed as a sum of an observed utility V_{ni} and an unobserved random disturbance ε_{ni} as in the following equation [12–15]:

$$U_{ni} = V_{ni} + \varepsilon_{ni} = \boldsymbol{\beta}_n^{\mathrm{T}} \mathbf{x}_{ni} + \varepsilon_{ni} \tag{1}$$

Here, $\boldsymbol{\beta}_n$ is a vector of part-worths for the n th individual, and $\mathbf{x}_{ni}$ is a vector of values describing the configuration of design alternative i. Usually, $\boldsymbol{\beta}_n$ is unknown and estimated statistically. For the unobserved random disturbance, ε_{ni}, there are many possible form choices. When using the standard normal distribution with i.i.d. (independent and identically distributed) assumption, the result is a probit model. When using a Gumbel distribution with the i.i.d. assumption, the outcome is a logit model because the difference between two extreme value distributions has a logistic distribution. Probit and logit models are nearly identical, except that the logit model has a slightly heavier tail [16]. Using the logit model, the choice probability that person n chooses an alternative i is obtained using the following equation [12]:

$$P_{ni} = \frac{e^{\boldsymbol{\beta}_n^{\mathrm{T}} \mathbf{x}_{ni}}}{\sum_j e^{\boldsymbol{\beta}_n^{\mathrm{T}} \mathbf{x}_{nj}}} \tag{2}$$

Preference heterogeneity is defined as a variation in taste across individuals [17] and leads to differentiated product specifications. The most widely used models to represent heterogeneity are the latent class multinomial logit (LC-MNL) model and the hierarchical Bayes mixed logit (HB-ML) model. The LC-MNL model was initially introduced as a way of formulating latent attitude variables from dichotomous survey items [18]. Since an assumption of this model is that latent variables are categorical, it has been extended to include nominal variables using a maximum likelihood algorithm [19]. The LC-MNL model first classifies individuals into several segments and then estimates segment-level part-worths. The preferences within each segment are estimated using the deterministic parameters of a multinomial model. Simultaneously, an individual's membership probability in each segment is estimated. In the design community, latent class analysis has been used by Besharati et al. [20], Williams et al. [21], and Turner et al. [22].

The HB-ML model defines individual-level preferences using continuous distribution functions. In a Bayesian inference of mixed logit models, model parameters are determined using multivariate distributions where $\beta \sim N(b, W)$. This model describes the probabilistic representation of choice behaviors by employing Bayesian inference for data augmentation and Markov-Chain Monte-Carlo (MCMC) methods to integrate over the parameter space. It is called a hierarchical model because there are two levels. The assumption at the higher level is that respondent preferences are normally distributed. At the lower level, a multinomial logit model is assumed to quantify choice probability [23]. These differences enable the HB-ML model to deal with stable individual-level results when respondents provide multiple observations. This model has been recently applied by Wang et al. [24], Shiau et al. [25], Foster et al. [26], Hoyle et al. [27], Kang et al. [28], and Michalek et al. [1].

While compensatory models using an additive utility rule have been widely used due to their simplicity, the same models also impose several limitations [12]. First, the additive utility rule may not accurately model real choice behavior because respondents often find it challenging to consider the entire set of product attributes when making a choice. Second, IIA (independence from irrelevant alternatives) becomes a challenge when using compensatory models because the choice probability of an alternative is affected by the presence of other alternatives. This issue arises from the i.i.d. assumption of the error term ε_{ni}. Some of these limitations are also inherent to Bayesian-based noncompensatory models, as discussed in Sec. 2.2.

2.2 Noncompensatory Choice Models. Researchers in economics and psychology have demonstrated that consumers use various heuristics to simplify their choice decisions [29]. By adopting heuristics, a two-stage decision process—referred to as a consider-then-choose model—has received attention because of its added realism. By employing noncompensatory screening rules, consumers narrow their decisions to a small set of products called a consideration set [10]. Then, they use a compensatory choice rule to evaluate the remaining products and make a selection.

Various heuristic decision rules for noncompensatory choices have been proposed, including conjunctive, disjunctive, lexicographic-by-aspects, elimination-by-aspects, and disjunctions of conjunctions (DOCs) [29]. The existing studies about Bayesian-based noncompensatory models [8,30] suggest that the conjunctive rule model is effective in both model fitness and predicting individual-level estimates. Hence, this article focuses on consider-then-choose models with the conjunctive screening rule, where consumers consider if the product has all "must have" and no "must not have" aspects [31]. It is formed by multiplying an indicator function across the attribute of an alternative as in the following equation [30]:

$$\prod_m \mathrm{I}(l_{im} > \gamma_m) = 1 \tag{3}$$

Here, l_{im} is the level of attribute m for choice alternative i. The cutoff value γ_m is the smallest level of the attribute that needs to be present for the consumer to consider the alternative [32]. The indicator function indicates whether or not a choice alternative is screened out in a noncompensatory choice. Thus, the indicator function $\mathrm{I}(\cdot)$ is equal to 1 when a level l_{im} exceeds a threshold value γ_m, and this indicates that the choice alternative i is not screened out. If the alternative has a lower level of the attribute than the cutoff value, the product is screened out.

Advances in Bayesian inference, machine learning, and greedoid languages make it possible to quantify consider-then-choose scenarios for a variety of heuristics. Noncompensatory models of conjunctive screening rules have also been applied to hierarchical Bayes multinomial probit (HB-MNP) models. The most significant difference from the HB-MNP is to additionally estimate the cutoff values in the upper level of the hierarchy, as in the following equation [8,30]:

$$P_{ni} = \mathrm{Prob}\left(U_{ni} > U_{nj} \text{ for all } j \text{ such that } \prod_m \mathrm{I}(l_{njm} > \gamma_{nm}) = 1 \right) \tag{4}$$

l_{njm} is the level of the attribute for respondent n for alternative j and attribute m. γ_{nm} is a respondent-level threshold of attribute m for respondent n. When an attribute is continuously distributed, it is assumed that the cutoff values are normally distributed. When an attribute consists of discrete levels, a multinomial distribution can be adopted such that $\gamma_{nm} \sim \text{Multinomial}(\boldsymbol{\theta}_m)$, where $\boldsymbol{\theta}_m$ is the vector of multinomial probabilities associated with the grid for attribute m. Each level is tested to determine the highest possible cutoff value (γ^*_{nm}) from allowable cutoff values (γ^a_{nml}) using the Metropolis–Hastings algorithm [12] based on a probability given in Eq. (5) [30] where l indicates attribute levels

$$\gamma_{nm} = \gamma^a_{nml} \text{ with probability } \frac{\mathrm{I}(\gamma^a_{nml})\theta_{ml}}{\sum_l \mathrm{I}(\gamma^a_{nml})\theta_{ml}} \tag{5}$$

This model returns an individual's part-worths, cutoff values, and cutoff probabilities each draw. Disjunctive and elimination-by-aspect rules can also be modeled using Bayesian inference [8,30].

The choice probabilities can be expressed as $(J-1)$-dimensional integrals over the differences between the errors because probit models are not closed form [12]. These differences are defined as $\tilde{V}_{nij} = V_{ni} - V_{nj}$ and $\tilde{\varepsilon}_{nij} = \varepsilon_{ni} - \varepsilon_{nj}$. Then, for the consider-then-choose process using a probit model, the choice probability that individual n chooses any alternative i that is in the consideration set is given by the following equation [30]:

$$P_{ni} = \begin{cases} \int \mathrm{I}(\tilde{V}_{nij} + \tilde{\varepsilon}_{nij} > 0 \;\; \forall j \neq i)\phi(\boldsymbol{\varepsilon}_n)d\boldsymbol{\varepsilon}_n & i,j \in C_n \\ 0 & i,j \notin C_n \end{cases} \tag{6}$$

$\mathrm{I}(\cdot)$ is an indicator of whether the statement in parentheses holds, $\phi(\boldsymbol{\varepsilon}_n)$ is the joint normal density with zero mean and covariance $\boldsymbol{\Omega}$, and C_n denotes a consideration set for consumer n. Noncompensatory attributes are used to determine whether a choice alternative is in a consideration set, and the remaining compensatory attributes are used in part-worth estimation.

The performance of noncompensatory models has been proven in terms of model fitness and predictability [8,30,32–35]. However, from the standpoint of design optimization, noncompensatory models have some inherent challenges:

- Inadequate screening rule assumptions may lead to an incorrect estimation of noncompensatory choices. Further, there is no general form to describe all noncompensatory heuristics. For example, the HB-MNP model with conjunctive screening rules can only describe choices that screen out attribute levels lower than the minimum requirements. This form may be inappropriate for nonincremental levels such as color or brand.
- Aggregate part-worths are difficult to use due to their probabilistic cutoff values. Each draw for a noncompensatory model results in a set of part-worths and cutoff values. Although MCMC can be used to consider all draws, this requires considerable computational power.
- Discontinuous choice probability functions (Eq. (6)) can cause numerical difficulty when precisely solving design constraints [9].

The most significant difference between compensatory and noncompensatory models is that cutoffs are estimated by determining the indicator value (Eq. (3)) at each MCMC draw to identify conjunctive rules. However, these indicator values are averaged across all respondents. Therefore, if aggregate part-worths are to be used to reduce computational expense, Eq. (4) cannot be directly used to predict choice probability. For compensatory models, however, respondent heterogeneity information can be maintained without the need for saving and using all draws.

Considering these challenges, compensatory models have numerous advantages from a product optimization perspective: (1) generalized forms, (2) draw information can be aggregated, and (3) likelihood functions are continuous. Further, when estimating part-worths at the individual-level, large absolute part-worth values (relative to the other part-worth values estimated in a zero-centered formulation) can cause the additive part-worth rule to act like a noncompensatory rule [29]. For these reasons, this article explores the challenges of using noncompensatory models and compares the results of a product optimization to those obtained when using compensatory models in the presence of noncompensatory choices.

3 Technical Approach

3.1 Synthetic Choice Data Generated by Virtual Agents. This paper explores how compensatory models can approximate a two-stage choice process, and examines the differences in an optimal design search when using compensatory and noncompensatory models. This approach is driven by the hypothesis that distinct population segments can be identified from the individual-level part-worths for specific attributes where noncompensatory decisions might be made. To verify this hypothesis, a two-stage process using the LC-MNL and HB-ML models is proposed, as shown in Fig. 1.

Discrete choice data are generated to mimic the consider-then-choose process, and are mathematically modeled using both noncompensatory and compensatory models. The HB-MNP model with a conjunctive screening rule is used as the noncompensatory model. The results of the cutoff distributions and posterior estimates show how the noncompensatory model describes the consider-then-choose process. To construct and understand the compensatory model, both LC-MNL and HB-ML models are used. A latent class analysis is first conducted to segment the population, and then the individual-level preferences are further explored based on the segmentation information. The latent class results are investigated to determine how attributes where

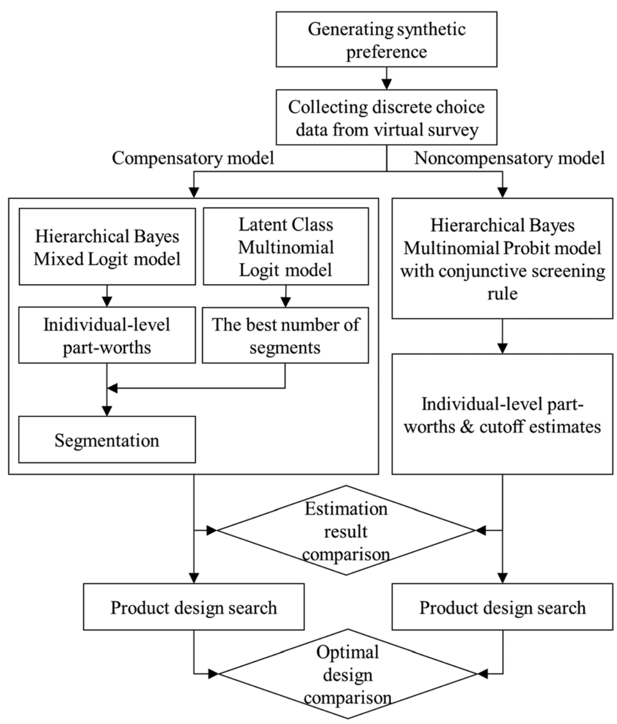

Fig. 1 Flowchart of the study used to compare compensatory models and a Bayesian-based noncompensatory model with conjunctive screening rules for synthetic choice data

noncompensatory decisions are made yield segments in terms of membership probability and attribute importance. Then, the obtained segments are evaluated using individual part-worths in terms of attribute importance and heterogeneity representation.

A product design problem is then solved using a genetic algorithm (GA) where individual-level preferences are used; the HB-ML and HB-MNP models are used as the compensatory and noncompensatory models, respectively. The design results are compared to show the suitability of compensatory modeling of the two-stage choice process for product search problems.

3.2 Real Choice Data Generated by Human Respondents. Choice data do not directly describe which respondents actually made noncompensatory choices, or what heuristics they used to make those choices. Thus, it is impossible to apply the same technical approach used to analyze the outcomes from synthetic choice data because "true" preferences are not available. For this reason, a different approach is proposed to assess and compare the optimum product designs of compensatory and noncompensatory models, as shown in Fig. 2.

Given part-worth estimates for both HB-ML model and HB-MNP model with conjunctive rules, optimum product design solutions can be obtained by maximizing the choice probability of the solution. Then, a hypothetical noncompensatory screening rule is assumed. To consider many noncompensatory choice scenarios, the disjunctions of conjunctions (DOCs) rule that generalizes conjunctive, disjunctive, and subset conjunctive rules [31] is used because the choice heuristics used by respondents are unknown. For an optimum product design, a consideration set verifier is simulated to calculate the likelihood (L_C) that the optimal solution is not screened out at the noncompensatory choice stage. This likelihood is obtained by dividing the number of feasible screening rules by the total number of choice simulations.

The need for a consideration set verifier when using choice task data is shown in Fig. 3. Assume that respondents took a discrete choice survey consisting of ten choice tasks involving products with two attributes and five total levels (three for attribute 1 and two for attribute 2). The data in Fig. 3 show the cumulative number of times each attribute level was chosen in the ten choice tasks. For product 1, created using level L3 of attribute A1 and level L1 of attribute A2, it is clear that this product was not screened out because the respondent chose these two attribute levels at least once when completing the choice tasks. However, for product 2, created using level L1 of attribute A1 and level L1 of attribute A2, the analysis is more complicated.

From the data available, it cannot be determined if the respondent screened out level L1 of attribute A1 using a noncompensatory screening rule, or if this attribute level was never chosen because of the compensatory tradeoffs that occur after the consideration set has been created. Hence, when running a consideration set verifier for several screening rules, the minimum likelihood that a product is not screened out can be estimated using a hypothetical noncompensatory choice simulation.

If it is assumed that a company introduces multiple new products into the market, the product line can be regarded as being included in the consideration set if any product in the line is not screened out. Thus, the general formula to calculate the likelihood value is obtained as

$$L_C = \frac{\sum_{N_{\text{ncs}}, N_{\text{resp}}} 1_{C_n}(\mathbf{x})}{N_{\text{ncs}} \times N_{\text{resp}}} \tag{7}$$

where N_{ncs} and N_{resp} indicate the number of noncompensatory scenarios and the number of respondents, respectively. The function $1_{C_n}(\mathbf{x})$ results in 1 if a product line design $\mathbf{x}$ is an element of a consideration set and 0 if $\mathbf{x}$ is not in a consideration set.

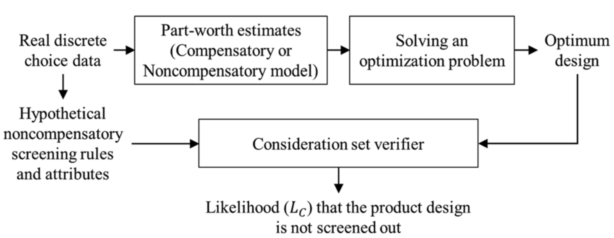

Fig. 2 Conceptual procedure for consideration set verifier using hypothetical screening rules

	A1			A2	
Level	L1	L2	L3	L1	L2
Cumulative number of choices	0	2	8	4	6

A: attribute, L: level

Consideration set verifier

- Product 1 = (L3 of A1, L1 of A2)
  Cumulative number of choices = (8,4)
  → Product 1 was not be screened out in noncompensatory choice
- Product 2 = (L1 of Attribute 1, L1 of Attribute 2)
  Cumulative number of choices = (0,4)
  → It cannot be figured out what happened for Product 2 in noncompensatory choice

Fig. 3 An example of a simulated noncompensatory choice using discrete choice data obtained from an actual survey

4 Case Study Using Synthetic Choice Data Generated by Virtual Agents

As shown in Sec. 3.2, choice data itself cannot be used to explicitly state respondents' choice processes. However, this information can be captured if synthetic data are created using predefined virtual agents. In this study, virtual respondents are generated using conjunctive screening rules because the existing studies of the Bayesian-based noncompensatory models suggest that the conjunctive rule model shows the best performance in both model fitness and predictability at individual-level estimates [8,30]. Synthetic choice data are collected using a simulated discrete choice survey. Part-worth estimates from the compensatory model and the noncompensatory model with conjunctive screening rules are obtained and compared. Finally, a product optimization is performed to compare differences in solution. Since only a conjunctive rule and a conjunctive model are used in this case study, the findings and discussions are limited to the conjunctive rule and its associated noncompensatory model. Exploring other forms of noncompensatory heuristic rules such as disjunctive, lexicographic-by-aspects, elimination-by-aspects, and disjunctions of conjunctions is a future research topic, as discussed in Sec. 6.

4.1 Generating Synthetic Choice Data. To generate synthetic survey data, a choice-based conjoint survey is designed around a vehicle selection scenario. Attributes and levels used in this study are described in Table 1. The manual transmission, automatic transmission without shift, and automatic transmission with shift are called MT, AT1, and AT2, respectively. TM is used

Table 1 Car attributes and levels used in virtual survey

	Transmission	Sunroof	A3	A4	A5	A6	Price
Level 1	MT	No	Two levels	Four levels	Four levels	Four levels	\$21,000
Level 2	AT1	Yes					\$20,000
Level 3	AT2						\$19,000
Level 4							\$18,000

Table 2 Predefined preferences of virtual respondents

Group	Number of respondents	Screen out	Must-have feature in consideration set
1	40	MT & no sunroof	AT1/AT2 & sunroof
2	40	MT	AT1/AT2
3	40	MT & AT1	AT2
4	40	No Sunroof	Sunroof
5	40	Only perform compensatory choices	

as an abbreviation of "transmission," and the capital letter A with a number is used to represent different product attributes. Survey questions are generated using Sawtooth SSI Web [36]. Respondents are asked to evaluate 16 buying scenarios and four holdout questions. Each scenario contains four product alternatives and a fifth no-buy option.

Table 2 shows the predefined preferences of the 200 virtual respondents used to form consideration sets at the first choice stage. Respondents use only the transmission and sunroof attributes when making noncompensatory choices, and lower levels are screened out. For example, respondents cannot screen out AT1 because the conjunctive rule assumes there is a minimum requirement value. If a respondent screens out AT1 only, the noncompensatory model with conjunctive rule cannot catch the behavior and the respondent is considered to make compensatory choices.

Respondents in groups 1–4 exhibit noncompensatory behavior and narrow their choice alternatives into a consideration set. Then, they compare all remaining alternatives and choose one. To mimic a real choice situation, if no alternative in the consideration set satisfies the minimum utility requirement, the no-buy option is selected. Respondents in group 5 perform only compensatory choices. To introduce heterogeneity, respondent preferences (excluding price) are generated based on uniform distributions with predefined intervals. Price preferences are manually generated and constrained so that respondents prefer lower prices. The virtual survey results in 3200 observations.

Attribute importance of the synthetic data for each group is shown in Table 3. Since attribute importance is calculated based on an additive rule assumption, the importance of noncompensatory attributes cannot be evaluated. However, maximum and minimum utility values of noncompensatory variables exist. These values are defined as

$$\max(V_{\mathrm{nc}}) + \sum_h \min(V_{c,h}) > V_{\mathrm{threshold}} > \min(V_{\mathrm{nc}}) + \sum_h \max(V_{c,h}) \quad (8)$$

Table 3 Attribute importance of the synthetic data

Group	TM	Sunroof	A3	A4	A5	A6	Price
1	50.0		4.2	9.6	10.1	9.6	16.6
2	50.0	4.7	5.0	8.2	8.8	8.5	14.9
3	50.0	5.7	4.2	8.6	9.6	7.5	14.4
4	7.1	50.0	4.8	8.0	8.3	7.8	14.0
5	11.7	8.1	8.7	14.9	15.1	16.1	25.3

where V is a part-worth set for each attribute, h indicates the number of compensatory attributes, while nc and c indicate noncompensatory and compensatory attributes, respectively. $\max(V_{\mathrm{nc}})$ indicates a part-worth of the noncompensatory attribute in the consideration set and $\min(V_{\mathrm{nc}})$ indicates a part-worth set of the noncompensatory attribute excluded from the consideration set. From Eq. (8), the smallest range of $\max(V_{\mathrm{nc}}) - \min(V_{\mathrm{nc}})$ is obtained as $\sum_h \max(V_{c,h}) - \sum_h \min(V_{c,h})$. Hence, the minimum attribute importance of a noncompensatory attribute is described by the following equation [37]:

$$\frac{\max(V_{\mathrm{nc}}) - \min(V_{\mathrm{nc}})}{\max(V_{\mathrm{nc}}) - \min(V_{\mathrm{nc}}) + \sum_h \{\max(V_{c,h}) - \min(V_{c,h})\}} \quad (9)$$

From this calculation, the minimum attribute importance of a noncompensatory attribute is 50%.

4.2 Noncompensatory Choice Modeling With Conjunctive Rules. The HB-MNP with conjunctive rule was fit using R [38]. Inference was conducted using Bayesian MCMC methods. The chain was run for the first 5000 iterations, with the final 5000 iterations used to estimate the moments of the posterior distributions.

Table 4 shows the aggregate estimates of the cutoff probability obtained using the conjunctive model. For discrete attributes, cutoffs are reported in terms of multinomial point mass probabilities. Each level is recorded as an integer (e.g., 0, 1, 2) and the recorded values indicate l_{nim} in Eq. (4). Thus, a grid of possible cutoff values, γ_{nm}, is also specified (e.g., −0.5, 0.5, 1.5, 2.5). The lowest cutoff value indicates that all levels are acceptable and that respondents made compensatory choices. The highest level indicates that none of the levels are acceptable [30]. A cutoff value of 0.5 indicates that only the lowest level is unacceptable, and 1.5 indicates that level 1 and 2 (recorded values 0 and 1) are screened out.

The probabilities of each cutoff obtained from the conjunctive model closely correspond with the predefined noncompensatory preferences in Table 2. For example, 40% of the total respondents were predefined to screen out MT in the virtual survey, and the conjunctive model results in 38% doing so. Estimates of cutoff probability equal to approximately 2% reflect the influence of the prior distribution and the inherent noise of MCMC. Therefore, respondents are shown to evaluate attributes 3, 4, 5, and price using a compensatory rule set.

For noncompensatory attributes, the part-worth estimates shown in Table 5 approach zero. This is because the choice probability is evaluated using only the alternatives in the consideration set. If an alternative is in the consideration set, then its choice probability is determined relative to the other alternatives in the

Table 4 Threshold estimates for the posterior means of the conjunctive model

Attribute	Level (recorded value)	Possible cutoff	Probability of each cutoff: Predefined (%)	Probability of each cutoff: Obtained (%)	Attribute	Level (recorded value)	Possible cutoff	Probability of each cutoff: Predefined (%)	Probability of each cutoff: Obtained (%)
TM	MT (0)	−0.5	40	38.0	A5	1 (0)	−0.5	100	88.6
	AT1 (1)	0.5	40	38.8		2 (1)	0.5	0	3.6
	AT2 (2)	1.5	20	20.6		3 (2)	1.5	0	2.6
		2.5	0	2.7		4 (3)	2.5	0	2.6
							3.5	0	2.6
Sunroof	No (0)	−0.5	60	58.1					
	Yes (1)	0.5	40	39.2	A6	1 (0)	−0.5	100	88.5
		1.5	0	2.8		2 (1)	0.5	0	3.6
						3 (2)	1.5	0	2.6
A3	1 (0)	−0.5	100	94.5		4 (3)	2.5	0	2.6
	2 (1)	0.5	0	2.8			3.5	0	2.6
		1.5	0	2.7					
					Price	$21,000 (0)	−0.5	100	87.6
A4	1 (0)	−0.5	100	89.2		$20,000 (1)	0.5	0	4.3
	2 (1)	0.5	0	3.0		$19,000 (2)	1.5	0	2.9
	3 (2)	1.5	0	2.6		$18,000 (3)	2.5	0	2.6
	4 (3)	2.5	0	2.6			3.5	0	2.6
		3.5	0	2.6					

set. If an alternative does not pass the screening rule, then its choice probability is zero. Thus, the screened alternative is not included in the posterior estimation processes. This leads to competition of alternatives in the consideration set that only has acceptable product features. Eventually, distinctions in posterior estimates between features are diminished and the estimates approach zero. In other words, noncompensatory attributes are excluded from part-worth estimation as explained in Sec. 2.2. Because of this, the part-worths of noncompensatory attributes, such as transmission and sunroof, are estimated to be relatively flat in comparison to the other attributes, as shown in Table 5.

4.3 Compensatory Choice Modeling

4.3.1 Latent Class Analysis. Segmentation of a population often occurs when respondents within a group have relatively similar preferences, but those preferences are quite different from group to group. It is hypothesized that if there are distinct attributes used to form consideration sets, these attributes will play the most significant role in defining different preferences from group to group. Latent class estimation was conducted using Sawtooth Software's CBC Latent Class module [39]. Statistical measures assess the goodness of fit [40], but often provide conflicting information on the optimal number of classes in the model. Based on the low rate of change in these statistics for models with five classes or more, the latent class model fit with five classes was selected.

Table 6 shows a comparison between the predefined respondent groups originally defined in Table 2 and latent class estimation. As listed in Table 2, the five groups are expressed using Arabic numbers according to their noncompensatory choices. The groups obtained from the latent class analysis, expressed using Roman numbers, are nearly identical to the predefined groups. In particular, all respondents in groups 1, 2, and 3 are placed in segments I, II, and III. Also, the 40 respondents in group 4, who were defined to screen the no-sunroof feature, are divided into segments I and IV. Since the "no sunroof" feature is screened out of segments I and IV, the two respondents moved from group 4 to segment I are still considered to maintain their preferences. The estimates of the respondents in group 5 depend on random preference generation and survey design. Even though these respondents did not make noncompensatory choices, if cumulative choices are rationally biased, it could be defined as a member of the segment that does make noncompensatory choices. This is because the latent class analysis does not estimate noncompensatory choices, but simply classifies respondents with similar preferences.

Membership probability demonstrates how effectively respondents are categorized into groups. Latent class estimation assumes that each respondent has some nonzero probability of belonging to each group. If the segmentation strategy fits the data very well, membership probabilities approach one. As shown in Table 7, respondents effectively have membership probabilities in only one class. The average maximum membership probability is 99.36%.

The attributes used to form consideration sets may be inferred from the attribute importance associated with each group. As shown in Table 8, the noncompensatory attributes associated with each segment have greater than 50% importance, similar to the result found in Eq. (9). Additionally, the attributes that result in the formation of consideration sets have much greater importance

Table 5 Part-worth estimates for the noncompensatory model

Attribute	Level	Posterior mean	Attribute	Level	Posterior mean
TM	AT1	0.09	A5	2	−0.30
	AT2	0.05		3	−0.26
				4	0.16
Sunroof	Yes	−0.07			
			A6	2	0.07
A3	2	0.40		3	−0.41
				4	−0.23
A4	2	0.34			
	3	0.32	Price	$20,000	1.78
	4	0.15		$19,000	3.31
				$18,000	5.08

Table 6 Number of members in each group

		Latent class: I	II	III	IV	V	Sum
Predefined group	1	40					40
	2		40				40
	3			40			40
	4	2			38		40
	5	1	4		10	25	40
	Sum	43	44	40	48	25	200

Table 7 Membership probability of belonging to a group

	Latent class				
Number of respondent	I	II	III	IV	V
43	99.88	0.00	0.00	0.12	0.00
44	0.99	98.68	0.00	0.00	0.33
40	0.00	0.01	99.99	0.00	0.00
48	0.00	0.00	0.00	99.96	0.04
25	0.00	1.67	0.00	0.01	98.31

Table 8 Attribute importance of latent class analysis

Segment	TM	Sunroof	A3	A4	A5	A6	Price
I	70.5		3.5	4.6	3.1	5.3	13.0
II	56.4	2.0	1.1	4.0	4.0	2.8	29.7
III	82.9	0.3	1.0	3.0	1.7	2.0	9.1
IV	3.4	50.7	2.6	5.1	4.9	2.5	30.8
V	5.7	11.6	5.1	9.9	10.4	9.1	48.2

Table 9 Part-worth estimates for the HB-ML model

Attribute	Level	Posterior mean	Attribute	Level	Posterior mean
TM	AT1	9.57	A5	2	−0.69
	AT2	13.04		3	−0.72
				4	−0.09
Sunroof	Yes	8.37			
			A6	2	0.56
A3	2	0.57		3	0.19
				4	0.16
A4	2	0.64			
	3	0.53	Price	$20,000	3.65
	4	0.17		$19,000	5.69
				$18,000	8.22

than the other attributes. Despite the strong inference about the attributes used in noncompensatory choice, latent class analysis does not explicitly identify if a noncompensatory screening is used. The result in Tables 7 and 8 shows that if there are noncompensatory choices, and the latent class results in estimates with high membership probabilities, importance for the attribute driving the noncompensatory rule will be higher than 50%.

4.3.2 Hierarchical Bayes Mixed Logit Model. While the number of segments and the features forming consideration sets can be speculated using the LC-MNL model, individual-level preferences can be estimated using the HB-ML model. This section provides detail on how the HB-ML model estimates mimic the two-stage choice process by presenting part-worths, attribute importance, and rank orders of the feature levels.

Aggregate zero-centered part-worth estimates for the HB-ML model are shown in Table 9. The HB-ML model was fit using the Sawtooth Software CBC/HB module [23]. For each respondent, 10,000 random draws were performed before averaging the next 10,000 random draws to create the posterior means. It is observable that the transmission and sunroof attributes have posterior means with larger deviations because they are used to mimic the behavior associated with creating the consideration sets. In contrast, the posterior means of the other attributes are relatively flat.

The results in Table 9 are for all respondents. Borrowing segmentation information from the latent class analysis, the individual-level part-worth estimates obtained from the HB-ML model are grouped by segment. For brevity, only the attributes used when making noncompensatory choices are listed. From these results, the hypothesis made in Sec. 3.1 that large absolute

Table 10 Hit rate comparison between HB-MNP with conjunctive rule and HB-ML models

Model	Hit rate (%)
HB-MNP with conjunctive rule	69.19
HB-ML	72.75

part-worth values (with respect to the other attributes) would be captured in the individual-level estimates is verified using the segment-level part-worths and the distributions of the individual-level part-worths.

A comparison of model performance using predictive accuracy is provided in Table 10. Predictive accuracy is defined by how well the model can predict a future set of observations. For synthetic choice data without added variability, a hit-rate measure also describes how well a model captures the predefined preferences of the virtual respondents. Using the four holdout questions, a hit-rate measure is obtained for each model. Hit rate is quantified using respondent-level aggregate part-worths for the compensatory model by averaging draw information. For the noncompensatory model, 500 draws were used because of the inability to calculate choice probabilities using aggregate part-worths as discussed in Sec. 2.2. Draw information could have been used for the compensatory model, and it would be expected that this would lead to a small change in predicted hit rate. However, this would have led to increased computational expense.

The results presented in Table 10 show that the compensatory model (HB-ML) has a slightly greater predictive accuracy than the noncompensatory model (HB-MNP with conjunctive rule). The original research paper presenting the noncompensatory model suggested that the HB-MNP model with conjunctive rule should have a greater predictive accuracy than a compensatory model (HB-MNP) [30]. However, this study demonstrates that a compensatory model can be more accurate in some scenarios, even though the comparison is between a logit model and a probit model. What is most significant is that using a compensatory model does not automatically introduce large prediction errors, even when noncompensatory decisions are being made.

This result could be caused by the inherent noise of MCMC for the cutoff estimation of the noncompensatory model discussed in Sec. 4.2. Although the hit rate difference is small, the larger hit rate for the HB-ML model is expected to reduce potential design errors due to incorrect preference estimation than the conjunctive rule model. However, this outcome is also influenced by holdout question design. Therefore, predictive accuracy measures alone cannot be used to generalize the performance of both models.

Table 11 shows that segments I–IV for the compensatory model have large absolute part-worth coefficients compared to the relatively flat part-worths estimated for segment V. The large absolute part-worths at the individual level are also observed in the histogram displayed in Fig. 4. The presence of large absolute part-worth values in a compensatory model is significant because if a part-worth value for an attribute is large enough, it can effectively mimic the upper stage of a noncompensatory screening rule. In the aggregate estimates of the HB-ML model, it is also noticeable that the AT1 feature of segment III is relatively flat. However, MT has a large absolute value despite the fact that the two features were screened out at the same time in the virtual survey. This is not a special case in the commercial software used; rather, it is likely an outcome of the prior distribution assumption.

All three histograms in Fig. 4 are closer to multimodal distributions than normal distributions. The distribution of heterogeneity has to be specified when estimating hierarchical Bayesian models, and a multivariate normal distribution is most commonly used. The commercial software used in this study also adopts the multivariate normal distribution. However, when the true distribution of heterogeneity is as close to a finite mixture of normal distributions as the noncompensatory choices, it is inappropriate to use a

Table 11 Zero-centered part-worth estimates of each segment obtained using the HB-ML model

Latent class	Transmission			Sunroof	
	MT	AT1	AT2	No	Yes
I	−12.7	5.6	7.1	−9.9	9.9
II	−10.9	5.5	5.4	−0.2	0.2
III	−11.7	−1.4	13.1	−0.5	0.5
IV	−0.4	−0.1	0.5	−8.6	8.6
V	0.1	−0.5	0.5	1.1	−1.1

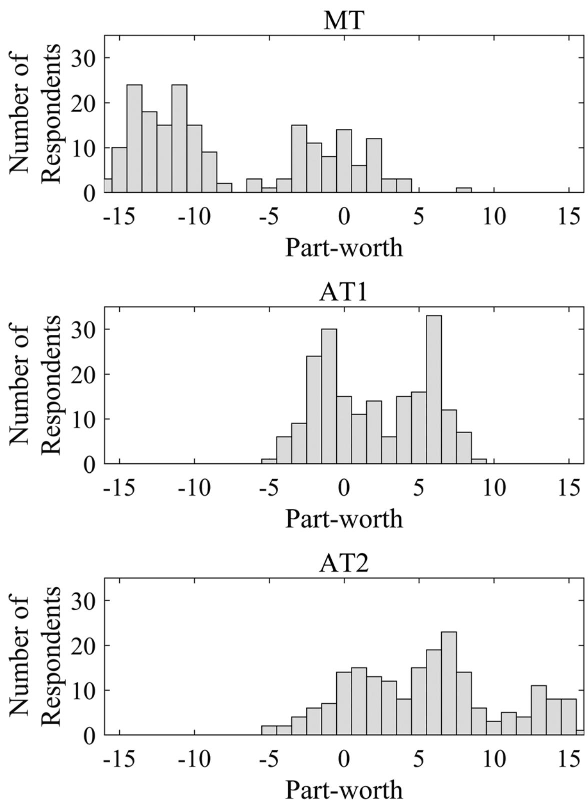

Fig. 4 Histogram of aggregate posteriors for transmission attribute obtained using the HB-ML model

multivariate normal. Thus, a hierarchical Bayes model may overestimate the proportion of the part-worths near zero [41]. Exploring appropriate prior distribution assumptions, or incorporating prior knowledge, is a source of future work.

Attribute importance values listed in Table 12 are obtained using the individual-level part-worths obtained from the HB-ML estimation. As discussed in Sec. 4.1, the noncompensatory variables have to have at least 50% importance; group 1 in the HB-ML model satisfies this condition. However, notice that groups 2, 3, and 4 contain importance values lower than 50%. This implies that the HB-ML model does not completely approximate a noncompensatory choice.

Importance values below 50% suggest a switching of products across the threshold of selection. For a respondent who screened out MT in each choice task, Fig. 5 depicts the switching of a product having an MT feature and the largest part-worths of each compensatory attribute. Even though the respondent never selected the MT feature in the virtual survey, some products having MT can be selected in a market simulation. This is due to the absence of a strict heuristic consideration rule in compensatory models. However, since product search problems only focus on the several top products, the impact of this scenario may be minimal.

Table 12 Attribute importance of HB-ML model

Group	TM	Sunroof	A3	A4	A5	A6	Price
1	67.9		2.8	3.7	6.1	4.9	14.5
2	41.2	7.1	6.0	7.2	8.8	9.0	20.6
3	43.7	8.3	5.2	8.2	9.6	8.6	16.4
4	11.5	42.2	5.6	6.5	7.3	6.9	20.0
5	14.8	11.8	8.0	10.7	13.6	13.6	27.6

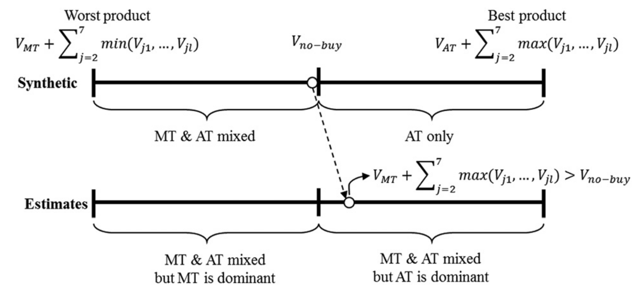

Fig. 5 Conceptual diagram to show the absence of a strict threshold in compensatory modeling of noncompensatory choice

The results in Table 13 help explain why the absence of a strict threshold may have minimal impact in optimization problems. Hundred and sixty respondents from groups 1 to 4 performed noncompensatory choices. Using the HB-ML part-worths, these screening rules can be reproduced for 150 of the 160 respondents. Further, the utility gaps between the best product and the switched product are significantly large. The effectively zero odds ratio values also suggest that the violated products likely have no have significant effect on a product search.

4.4 Product Design Search.

This section focuses on product configuration differences when using the compensatory and noncompensatory models in an optimization. Individual-level aggregate estimates of the HB-ML model are used as the compensatory model. In contrast, as explained in Sec. 2.2, using the aggregate values of the conjunctive model is challenging because of the probabilistic representation of cutoffs and corresponding part-worth estimates. For this reason, 5000 draws were generated using a MCMC process. To ensure independency between draws and to manage the computational cost of this procedure, every tenth draw was kept for use in the optimization, leading to 500 draws per individual. All draws were equally weighted when evaluating share of preference.

The pricing structure for each attribute is shown in Table 14. In addition to this pricing structure, a base price of $18,000 is added. A piecewise linear interpolation is used to calculate the price attribute part-worth. The objective of the search is maximizing the choice probability of the product configuration in a competitive market, using Eqs. (2) and (6) for the compensatory and noncompensatory models, respectively. Competitor products are defined in Table 15.

The first scenario considered was finding the best product to offer. This scenario does not necessarily require a search algorithm because only 768 product configurations had to be evaluated. The second scenario considered was finding the optimal configurations when offering two products, leading to a problem size of 589,056 product combinations. This required a search algorithm because one market simulation using the noncompensatory model took approximately 15 s on a laptop running an Intel i7 2.20 GHz with 16 GB RAM. A genetic algorithm was used for both model formulations because a GA works directly with discontinuous choice probabilities and previous work has shown

Table 13 Part-worth comparison of the switched products

Predefined group	No. of respondents having switched products	Avg. utility of the best product[a]	Avg. utility of the best product among switched products[b]	Threshold ($V_{\text{no-buy}}$)	Odds ratio[c]
2	4	16.3	1.5	1.0	$\cong 0$
3	2	20.7	8.7	8.0	$\cong 0$
4	4	22.5	4.0	1.8	$\cong 0$

[a] $V_{\text{nc,consideration}} + \sum \max(V_{c1}, \ldots, V_{cl})$.

[b] $V_{\text{nc,screen out}} + \sum \max(V_{c1}, \ldots, V_{cl})$.

[c] $\frac{\exp\left(V_{\text{nc,screen out}} + \sum \max(V_{c1},\ldots,V_{cl})\right)}{\exp\left(V_{\text{nc,consideration}} + \sum \max(V_{c1},\ldots,V_{cl})\right)}$.

Table 14 Pricing structure (in $)

	TM	Sunroof	A3	A4	A5	A6
Level 1	0	0	0	0	0	0
Level 2	800	500	500	100	200	100
Level 3	1000			200	300	200
Level 4				300	400	300

Table 15 Attribute levels of competitor products in the market

Competitor	TM	Sunroof	A3	A4	A5	A6	Price
Product 1	MT	No	1	1	1	1	\$18,000
Product 2	AT1	Yes	1	2	3	3	\$19,900
Product 3	AT2	Yes	2	4	4	4	\$21,000

the advantages of this technique in a product line optimization [22,26]. The pool size was set at 200 and the stall generation limit was set to 50.

Solutions to the two optimization scenarios are shown in Tables 16 and 17, respectively. Investigating solution differences is divided into two aspects—the inclusion of must-have features/attributes and design error. Design error is quantified by evaluating the objective function using both the synthetic (true) preferences generated in Sec. 4.1 and the estimated preferences from the model fits. SOP_o indicates the share of preference used as an objective function value. SOP_t indicates the share of preference evaluated in the synthetic (true) preferences. The design error metric is defined as (SOP_t of synthetic data— SOP_t of estimated data)/SOP_t of synthetic data. For instance, the design error of the compensatory model for the scenario 1 is obtained as $(37.1 - 35.9)/37.1 = 3.2\%$. Notice that the gap, $\text{SOP}_t - \text{SOP}_o$, does not directly provide an evaluation of a design solution because the value can change by adjusting the scale parameter σ_β and other settings associated with model estimation.

In the one product design scenario, the noncompensatory attributes (transmission and sunroof) are represented by the levels used to form the consideration sets. The compensatory model product configuration has three of the correct compensatory features, while the noncompensatory model product configuration only has one correct. These results imply that if there are strong noncompensatory choices, the noncompensatory attributes can be found regardless of model. In terms of design error, the compensatory model performs better than the noncompensatory model.

Similar outcomes are observed in the two-product scenario. The true data and the compensatory model use the same transmission and sunroof features, though there is discrepancy in some of the compensatory attributes. Although the noncompensatory model resulted in only AT2 for the transmission feature, the crucial finding is that both models find solutions that would be included in the consideration sets.

The predictive accuracy listed in Table 10 provides evidence that supports why the compensatory model (HB-ML) resulted in smaller design errors than the conjunctive model (HB-MNP with conjunctive rule) in both problems. The slighter greater predictive accuracy of the compensatory model implies that the compensatory model better reflects the true preferences of the respondents. However, this result is limited to the simulated data for this study and additional research is needed to understand design problem formulation influences this result.

For the compensatory attributes, there exist both commonality and discrepancy in the optimal product configurations. To

Table 16 Optimal product configuration for each model (scenario 1)

Model	TM	Sunroof	A3	A4	A5	A6	Price	SOP_o (%)	SOP_t (%)	Design error (%)
True	AT2	Yes	1	3	1	2	\$20,100	37.1	37.1	—
Compensatory	AT2	Yes	1	2	1	2	\$20,300	52.4	35.9	3.2
Noncompensatory	AT2	Yes	1	1	3	3	\$20,350	41.3	29.2	21.3

Table 17 Optimal product configuration for each model (scenario 2)

Model	TM	Sunroof	A3	A4	A5	A6	Price	SOP_o (%)	SOP_t (%)	Design error (%)
True	AT1	Yes	1	1	3	3	\$19,850	54.2	54.2	—
	AT2	Yes	1	2	1	2	\$20,300			
Compensatory	AT1	Yes	2	2	1	3	\$19,900	68.7	47.8	11.8
	AT2	Yes	1	2	1	2	\$20,300			
Noncompensatory	AT2	Yes	2	1	3	3	\$20,550	61.8	45.6	15.9
	AT2	Yes	1	4	2	4	\$20,400			

Table 18 Choice probability interval sensitivity study

Data	Interval between max. and min. choice probabilities (%)					
	TM	Sunroof	A3	A4	A5	A6
True	19.2	9.5	0.6	3.4	5.3	7.2
Compensatory	36.6	22.2	1.4	6.6	16.4	9.1
Noncompensatory	36.7	18.2	1.8	4.3	7.9	9.7

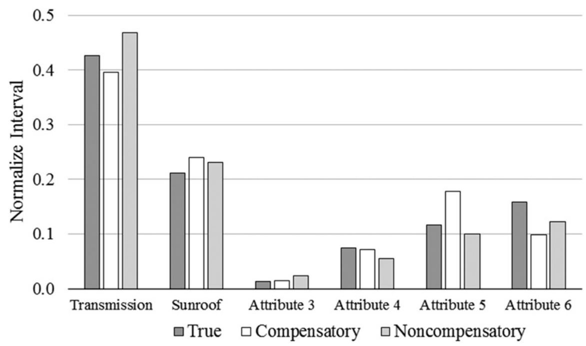

Fig. 6 Interval comparison between the max. and min. choice probabilities of each attribute

investigate how many differences exist between the three solutions, choice probabilities are tested at all attribute levels to assess the sensitivity of the share of preference calculation. The intervals between the maximum and minimum choice probabilities when only one attribute changes its level are shown in Table 18.

The results in Table 18 show that, for both models, the intervals between maximum and minimum choice probabilities are larger than the true data. Also, the noncompensatory attributes have significantly larger intervals than the compensatory attributes. Normalized intervals of choice probability for each model are displayed in Fig. 6. This result is analogous to attribute importance. As a general result, the noncompensatory model tends to give more weight to the noncompensatory attributes while underweighting the compensatory attributes. The result for the compensatory model is less structured, with variability across all attribute types.

In this study, the compensatory model is more accessible because of the reduced computational burden and no requirement for screening rules. Product configurations from both estimated models found solutions very similar to the optimal solution when the true preferences are used. When calculating design error, the compensatory model outperformed the noncompensatory solution in both the one- and two-product design scenario. To further support this finding, Sec. 5 explores a problem using choice data from real respondents.

5 Case Study Using Real Choice Data Generated by Human Respondents

5.1 Survey Design and Modeling. To explore solution differences when using choice data from an actual survey, a second case study was conducted. Beyond the motivation listed in Sec. 3.2, a study involving human responses was pursued to ensure that the results in Sec. 4 were not an artifact associated with the generation of the synthetic data. A discrete choice survey with 12 choice tasks was completed by 205 respondents. For each question, a respondent was faced with four MP3 player configurations and a "No-Buy" option. Respondents were then asked to choose one product alternative they would be most likely to purchase. Each MP3 player was composed of eight product attributes. Product attributes and their levels are listed in Table 21 of the Appendix.

Once the survey data were collected, the HB-MNP with a conjunctive rule was fit using R [38], and the HB-ML was fit using Sawtooth Software's CBC/HB module. For both models, 10,000 random draws were used for each respondent before averaging the next 10,000 random draws to estimate the moments of the posterior distributions. For the noncompensatory model, every 20th draw was kept for use in the optimization to ensure draw independency and to manage the computation expense. This resulted in 500 draws per individual.

Table 22 in the Appendix shows the aggregate estimates of the cutoff probability obtained using the conjunctive model. Conjunctive screening rules were estimated at 55.2% and 75.7% for the storage size and price attributes, respectively. They can be regarded as incremental attributes whose higher levels are preferred. This trend is shown as large absolute part-worth values of the compensatory model as shown in Table 23. In addition, this leads to the relatively large attribute importance values in the compensatory model, as listed in Table 24. This suggests that even though the compensatory model does not have an ability to capture noncompensatory choice behavior, biased preference in the noncompensatory attributes is maintained.

5.2 Product Design Search. Individual-level aggregate estimates of the HB-ML model are used for the compensatory model. In contrast, simulations involving 500 draws were used for each respondent in the noncompensatory model. Three competitor products are defined. The objective of the search is to maximize the choice probability of the optimized product configuration (or line) in a competitive market, using Eqs. (2) and (6) for the compensatory and noncompensatory models, respectively. The consideration set generator used in this study assumes respondents focus on only a small subset of product attributes in the noncompensatory choice stage to simplify their choice decisions. Combinations of conjunctive and disjunctive rules are assumed in the simulation. The maximum number of subset conjunctive and disjunctive rules is set as two and one. In set theory, the two subset conjunctive rules and one disjunctive rule are expressed as $(C_1 \cap C_2) \cup D_1$, where C and D indicate conjunctive and disjunctive rules, respectively. Thus, 332 different noncompensatory choice scenarios exist, which is obtained as $({}_8C_2 + {}_8C_1 + {}_8C_0) \times ({}_8C_1 + {}_8C_0) - 1$. By averaging the results of the 332 choice simulations, the minimum likelihood (L_C) that a product design is not screened out in the consideration set verifier can be estimated using Eq. (7).

The first scenario was to find the best product to offer, leading to 393,216 possible product feature combinations. The second scenario considered was finding the optimal configurations when offering two products, leading to a problem size of about 1.54×10^{11} product combinations. A genetic algorithm was used for both problems. The pool size was set as ten times ndv, where ndv indicates the number of design variables and the stall generation limit was set to 100.

Solutions to the two optimization scenarios are shown in Tables 19 and 20, respectively. The L_C value was obtained by evaluating the optimum design using the consideration set verifier for the 332 scenarios. SOP_C and SOP_{NC} are used to show the share of preference when evaluating a design using part-worths data associated with the compensatory and noncompensatory models, respectively. In addition to the optimal product configurations from each model, the choice task data were mined to identify the design with the maximum L_C value for comparative purposes.

A noticeable result from both scenarios is that the likelihood values, L_C, are similar across all three cases—compensatory model, noncompensatory model, and choice task mining. For the single-product scenario, 59.3% is the maximum likelihood value that can be achieved, and this number increases to 72.7% for the two product search. It is also important to note that maximizing L_C using only choice task mining does not guarantee a high SOP value because it is unable to model the lower phase (choose) of the consider-then-choose process.

Table 19 Optimal product configuration for each model (scenario 1)

Data	A1	A2	A3	A4	A5	A6	A7	Price	SOP_C (%)	SOP_{NC} (%)	L_C (%)
Aggregate part-worths (compensatory model)	5	8	3	3	2	1	4	\$209	33.8	25.5	49.4
Draws from noncompensatory model	8	8	3	4	2	3	4	\$246	20.8	29.5	51.9
Choice task mining (max. L_C)	3	5	3	3	2	6	1	\$192	4.0	17.8	59.3

Table 20 Optimal product configuration for each model (scenario 2)

Data	A1	A2	A3	A4	A5	A6	A7	Price	SOP_C (%)	SOP_{NC} (%)	L_C (%)
Aggregate part-worths (compensatory model)	5 8	8 5	3 3	3 4	2 5	1 8	4 3	\$209 \$396	50.3	42.0	65.7
Draws from noncompensatory model	5 8	8 5	2 3	4 5	2 5	3 6	4 3	\$194 \$396	32.5	45.9	66.7
Choice task mining (max. L_C)	3 6	5 6	3 4	3 6	2 6	7 6	1 4	\$192 \$413	6.5	24.7	72.7

To further explore the results presented in Tables 19 and 20, a comparison of product price is appropriate because it is the strongest noncompensatory attribute, as shown in Table 22. The optimum product price in the single product search for both models (\$209 and \$246) are similar to the price obtained when maximizing L_C (\$192) after mining the discrete choice data. Also, for the two product search shown in Table 20, one product is around \$200 and another is around \$400. This supports the hypothesis that solutions developed using compensatory models find solutions similar to those generated when using noncompensatory models, and that these solutions will not be screened out by consumers making noncompensatory choices.

The results in Table 20 also demonstrate that for a two-product solution, the optimal product solutions are quite similar in configuration. Differences in product configuration occur in attributes that are among the least important (A3 and A6). These attributes also pose a challenge for noncompensatory screening as the order in which the attribute levels should be presented to ensure proper screening is not easily apparent, nor does it have particular meaning. Significantly, when the solution associated with the compensatory model is evaluated using the noncompensatory draws, the estimated share of preference is only a few percentage points less than the solution obtained using the noncompensatory model. This suggests that a compensatory model can be advantageous and useful even if a product designer believes noncompensatory choices have been made because of the challenges associated with noncompensatory models discussed in Sec. 2.2. Without strict screening rules, it is possible for a compensatory model to mimic the two-stage choice process using large absolute part-worth values to prevent product solutions with screened out product attributes.

6 Conclusions

The main purpose of this paper was to explore the suitability of using compensatory models to mimic the consider-then-choose process when trying to design an optimal product offering. This work is motivated by the potential errors associated with assuming screening rules, probabilistic representations of noncompensatory choices, and discontinuous choice probability functions associated with existing Bayesian-based noncompensatory models. It was hypothesized that distinct segments would be captured where screening occurred, and that large absolute part-worth values would be found in the individual-level estimates of the HB-ML model.

To verify this hypothesis, this research first investigated segmentation techniques of the two-stage choice process using latent class analysis. Using latent class is based on the idea that noncompensatory choices would cause a distinct differentiation of population preference. The numerical results of latent class analysis confirm this hypothesis. The distribution of preference heterogeneity is explored to compare the true preference and the compensatory model at individual-level preference. The results of the individual-level preference analysis show that the HB-ML model can represent noncompensatory choices using large absolute values in part-worths despite the absence of strict thresholds. Lastly, implications of model choice between the two representations of the consider-then-choose process are discussed using the results of the product design search problem. The results of the product design search show interesting implications of model form choice. Although there are several insignificant differences between the two models in the market simulation, the compensatory model has some significant advantages such as the small design error and its relatively inexpensive computational burden. In the analysis of the real choice data, results also confirm the suitability of the compensatory model in product design search as its optimum design has an acceptable level in the likelihood gap associated with the proposed consideration set verifier.

A limitation of this work is that the attributes used to make noncompensatory choices may not have the largest importance in a latent class model when only a small number of respondents make noncompensatory choices. However, this should not be a concern when searching for an optimal solution as the outcome reflects solutions capable of maximizing or minimizing an objective across all respondents. In this case, the small number of the respondents performing noncompensatory choices would not have a significant effect on the optimization problem. Further, a standard choice-based study was used. These findings should be explored using more complex survey tools like adaptive choice that refines the choice alternatives seen by a respondent as data from the choice tasks are gathered. Finally, it should be noted that the findings of this work cannot be generalized to all noncompensatory choices and all noncompensatory models that exist in the literature. Rather, this study was focused on conjunctive screening rules estimated using a Bayesian-based noncompensatory model.

Future work will focus on seven different challenges: (1) developing optimization techniques capable of using the compensatory model of the two-stage choice process, (2) resolving the limitation of assuming normally distributed priors, (3) exploring differences between compensatory models when different noncompensatory heuristics are used by respondents, (4) investigating the implications of the discrepancy between the true preference distribution and the prior distribution assumption of Bayesian inference, (5) exploring the effect of modeling other noncompensatory heuristic rules with a compensatory model in a product design search, (6) developing segmentation methods for noncompensatory models, and (7) generalizing the observation in this paper by quantifying asymptotic performance of each model.

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Subscripts

c = compensatory
h = number of compensatory attributes
i = choice alternative
j = choice alternatives in a choice scenario
k = choice scenario
l = attribute level
m = attribute
n = respondents
nc = noncompensatory

Appendix: MP3 Survey Data and Modeling Result

Table 21 MP3 attributes and levels

	Attributes							
	A1	A2	A3	A4	A5	A6	A7	
Level	Photo/video/camera	Web/app/ped	Input	Screen size	Storage size	Background color	Background overlay	Price
1	None	None	Dial	1.5 in. diag.	2 GB	Black	No pattern/graphic overlay	$699
2	Photo only	Web only	Touch-pad	2.5 in. diag.	16 GB	White	Custom pattern overlay	$599
3	Video only	App only	Touch-screen	3.5 in. diag.	32 GB	Silver	Custom graphic overlay	$499
4	Photo and video only	Ped only	Buttons	4.5 in. diag.	64 GB	Red	Custom pattern and graphic overlay	$399
5	Photo and lo-res camera	Web and app only		5.5 in. diag.	160 GB	Orange		$299
6	Photo and hi-res camera	App and ped only		6.5 in. diag.	240 GB	Green		$199
7	Photo, video and lo-res camera	Web and ped only			500 GB	Blue		$99
8	Photo, video and hi-res camera	Web, app, and ped			750 GB	Custom		$49

Table 22 Threshold estimates for the posterior means of the conjunctive model (MP3 data)

Attribute	Level (recorded value)	Possible cutoff	Probability of each cutoff (%)	Attribute	Level (recorded value)	Possible cutoff	Probability of each cutoff (%)
A1	1 (0)	−0.5	65.2	A5	1 (0)	−0.5	44.8
	2 (1)	0.5	12.7		2 (1)	0.5	27.5
	3 (2)	1.5	4.4		3 (2)	1.5	8.1
	4 (3)	2.5	3.7		4 (3)	2.5	6.0
	5 (4)	3.5	2.7		5 (4)	3.5	3.9
	6 (5)	4.5	3.6		6 (5)	4.5	2.6
	7 (6)	5.5	2.8		7 (6)	5.5	2.4
	8 (7)	6.5	2.4		8 (7)	6.5	2.4
		7.5	2.4			7.5	2.4
A2	1 (0)	−0.5	56.2	A6	1 (0)	−0.5	73.3
	2 (1)	0.5	20.7		2 (1)	0.5	7.0
	3 (2)	1.5	5.6		3 (2)	1.5	3.5
	4 (3)	2.5	4.1		4 (3)	2.5	3.6
	5 (4)	3.5	3.3		5 (4)	3.5	3.0
	6 (5)	4.5	2.6		6 (5)	4.5	2.4
	7 (6)	5.5	2.4		7 (6)	5.5	2.4
	8 (7)	6.5	2.7		8 (7)	6.5	2.4
		7.5	2.4			7.5	2.5
A3	1 (0)	−0.5	83.8	A7	1 (0)	−0.5	87.6
	2 (1)	0.5	5.5		2 (1)	0.5	4.0
	3 (2)	1.5	5.4		3 (2)	1.5	3.1
	4 (3)	2.5	2.6		4 (3)	2.5	2.7
		3.5	2.7			3.5	2.7
A4	1 (0)	−0.5	64.0	Price	$699 (0)	−0.5	24.3
	2 (1)	0.5	15.6		$599 (1)	0.5	13.7
	3 (2)	1.5	9.5		$499 (2)	1.5	12.4
	4 (3)	2.5	3.4		$399 (3)	2.5	14.6
	5 (4)	3.5	2.4		$299 (4)	3.5	10.8
	6 (5)	4.5	2.6		$199 (5)	4.5	12.4
		5.5	2.5		$99 (6)	5.5	6.2
					$49 (7)	6.5	3.1
						7.5	2.4

Table 23 Part-worth estimates of compensatory and noncompensatory models (MP3 data)

		Posterior mean				Posterior mean	
Attribute	Level	Attribute	Level	Compensatory	Noncompensatory	Compensatory	Noncompensatory
A1	2	1.20	−1.13	A5	2	5.21	1.00
	3	1.50	−0.53		3	5.10	0.31
	4	2.31	−0.29		4	6.85	1.57
	5	3.04	0.83		5	7.76	1.82
	6	3.19	0.58		6	6.45	0.99
	7	2.42	0.29		7	7.27	2.37
	8	4.88	2.49		8	6.94	1.46
A2	2	4.14	1.40	A6	2	−0.72	−0.75
	3	2.97	−0.35		3	−0.05	0.39
	4	0.63	−2.20		4	−0.52	−0.54
	5	6.24	3.11		5	−1.09	−1.17
	6	3.70	0.31		6	−1.07	−0.60
	7	5.25	1.88		7	−1.05	−1.47
	8	6.73	3.53		8	−1.03	−0.95
A3	2	−1.01	−0.37	A7	2	0.32	0.13
	3	2.57	2.43		3	1.00	0.52
	4	−0.34	−0.47		4	1.07	0.71
A4	2	1.25	−0.08	Price	\$599	−0.51	−2.87
	3	3.69	0.64		\$499	4.52	0.13
	4	3.94	1.36		\$399	5.88	−0.87
	5	3.35	1.07		\$299	8.92	1.20
	6	4.12	1.43		\$199	14.07	4.26
					\$99	15.92	5.16
					\$49	18.13	6.46

Table 24 Attribute importance of compensatory model (MP3 data)

	A1	A2	A3	A4	A5	A6	A7	Price
Average	10.8	14.7	8.6	9.9	14.4	7.4	4.4	29.8
Maximum	23.0	27.3	26.7	22.6	23.7	18.3	10.9	53.1
Minimum	3.3	3.7	0.9	1.8	5.9	2.2	0.3	8.7

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