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## **MODELING NONCOMPENSATORY CHOICES WITH A COMPENSATORY MODEL FOR A PRODUCT DESIGN SEARCH**

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### **ABSTRACT**

Market-based product design has typically used compensatory models that assume a simple additive part-worth rule. However, marketing literature has demonstrated that consumers use various heuristics called noncompensatory choices to simplify their choice decisions. This study aims to explore the suitability of compensatory modeling of these noncompensatory choices for the product design search. This is motivated by the limitations of the existing Bayesian-based noncompensatory mode, such as the screening rule assumptions, probabilistic representation of noncompensatory choices, and discontinuous choice probability functions in the Bayesian-based noncompensatory model. Results from using compensatory models show that noncompensatory choices can lead to distinct segments with extreme part-worths. In addition, the product design search problem suggests that the compensatory model would be preferred due to small design errors and inexpensive computational burden.

Keywords: market-based product design; product line optimization; discrete choice model; compensatory model; noncompensatory choice; hierarchical Bayesian

### **1. INTRODUCTION**

In a globally competitive market, it is a significant challenge for a company to understand consumer preferences and introduce desired products. Market-based product design optimization provides methods to aid product design so that a diverse set of customer segments can be satisfied. As one of the tools for estimating customer preferences, discrete choice analysis has been widely used by implementing a variations of the generalized linear models such as the logit and probit

models from the market research community [1–4].

Many forms of the discrete choice models used in market-based design assume that consumers make compensatory choices. Compensatory choices are based on an additive utility rule; that is, high levels on some features can compensate for low levels on other features. However, a number of papers have demonstrated that noncompensatory choice models often improve both model realism and accuracy in predicting consumers' choices [5–8]. The noncompensatory choice rule supposes consumers' choice tasks are conducted using heuristic decision-making strategies. Imagine a consumer, who does not want a manual transmission, shopping for a new car. The consumer first narrows his choices to a small set of cars equipped with an automatic transmission and then compares the cars in the set to select one using the additive rule. This choice behavior is called a consider-then-choose process. The first stage forms a consideration set, which is called the noncompensatory choice. The second stage compares product alternatives and selects one in the set, which is called the compensatory choice. Under the consider-then-choose process, one of the most significant challenges is to launch products that are not being screened out of the consideration set.

Since the early 2000s, there has been increased development in modeling noncompensatory choices using computationally expensive methods like Bayesian inference and machine learning techniques. However, the effectiveness of noncompensatory models in a product design search has not yet been entirely explored. Morrow et al. proposed nonlinear programming relaxations for market-system design optimization problems to deal with the discontinuous likelihood functions of a consider-then-choose model [9]. Long and Morrow investigated the impact of noncompensatory choice

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behavior on optimal design for compensatory models [10]. Their studies, however, have been focused on evaluating predictive power and design error at the population level without exploring individual-level part-worth estimates.

The objectives of this paper are to investigate the suitability of the using Bayesian-based noncompensatory models to estimate individual-level choice behaviors for a product design search and compare the results of a product optimization to the results using a compensatory hierarchical Bayes mixed logit model. This study is motivated by the inherent disadvantages of inadequate screening assumptions, probabilistic representations of the noncompensatory choices, and discontinuous choice probability functions that accompany noncompensatory models. The suitability of using existing noncompensatory models is discussed in Sections 2 and 3 by reviewing the existing models. This concept is also examined in Section 4 by analyzing synthetic data. In particular, Section 3 addresses the limitations of an existing noncompensatory model and explores the performance of using a compensatory model in the presence of noncompensatory choices. Finally, a product optimization is conducted using both the compensatory and noncompensatory models and their results are compared.

## 2. BACKGROUND

The necessary background knowledge about discrete choice models is introduced to aid in the explanation of this study. Section 2.1 briefly reviews the fundamental concepts of compensatory models that have been typically used in market-based design - latent class multinomial logit (LC-MNL) and hierarchical Bayes mixed logit (HB-ML). In Section 2.2, various heuristics of noncompensatory choices and their modeling methods are reviewed, mainly focusing on the HB multinomial probit model with conjunctive screening rule.

### 2.1 Compensatory choice models

In market-based product design, consumer preferences have typically been modeled using compensatory models. The assumption behind compensatory models is that consumers weigh and compare all available attributes of products in the market before selecting a product. Discrete choice analysis is used to model product demand by capturing a customer's choice behavior [11]. It is a quantitative process for creating a demand model based on the collected data about a customer's preferences. The basic assumption for the analysis is that each customer's choice in product selection is governed by a utility function which includes an unobservable quantity.

The choice utility that person  $n$  obtains from alternative  $j$  can be expressed as a sum of an observed utility  $V_{nj}$  and an unobserved random disturbance  $\varepsilon_{nj}$  as [12–15]

$$U_{nj} = V_{nj} + \varepsilon_{nj} = \beta_n^T x_{nj} + \varepsilon_{nj} \quad (1)$$

where  $\beta_n$  is a vector of part-worths for  $n$  th individual and  $x_{nj}$  is a vector of values describing the  $j$  th design alternative in that choice task. Usually,  $\beta_n$  is unknown to the researcher and is estimated statistically. For the unobserved random disturbance  $\varepsilon_{nj}$ , there are many possible choices, but it is

usually considered as either the standard normal distribution function or a Gumbel distribution function. When using the standard normal distribution function, the resultant model is called the probit model. For the second case, using the Gumbel distribution, the resultant model is called a logit model because the difference between two extreme value distributions has a logistic distribution. The probit and logit models are nearly identical, except that the logit model has a slightly flatter tail [16]. Using the logit model, the choice probability that person  $n$  chooses an alternative  $i$  can be obtained from the simple logit formula as [12]

$$P_{ni} = \frac{e^{\beta_n^T x_{ni}}}{\sum_j e^{\beta_n^T x_{nj}}} \quad (2)$$

Preference heterogeneity is defined as a variation in taste across individuals [17] and leads to differentiated product specifications. Thus, understanding the diversity of consumer preferences plays a central role in the early stages of the product development process. Recently, models that represent preference heterogeneity have become more prevalent.

The most widely used models to represent heterogeneity are the latent class multinomial logit (LC-MNL) model and the hierarchical Bayes mixed logit (HB-ML) model. The LC-MNL model was initially introduced as a way of formulating latent attitude variables from dichotomous survey items [18]. Since this model assumes that the latent variables are categorical, it has been extended to the discrete representation of heterogeneity by including nominal variables and integrating the maximum likelihood algorithm [19]. To capture market heterogeneity, the LC-MNL model classifies respondents into segments with similar preferences. This approach is to subdivide a sample of respondents into discrete segments and to estimate the preferences within each segment using the deterministic parameters of a multinomial model. In the design community, latent class analysis has been as a tool for market segmentation by Besharati et al. [20], Williams et al. [21], and Turner et al. [22].

The HB-ML model exemplifies a continuous representation of heterogeneity because the mixed logit model is used to define individual-level preferences using continuous distribution functions. In multinomial logit models, it is basically assumed that data is described by a particular model with fixed parameters. In contrast, in a Bayesian inference of mixed logit models, the parameters of the model are determined based on multivariate distributions by setting up as  $\beta \sim N(b, W)$  with parameters  $b$  and  $W$  that are estimated. In other words, this model describes the probabilistic representation of choice behaviors by employing the Bayesian inference for data augmentation and Markov-Chain Monte-Carlo methods to integrate over the parameter space. Thus, it requires minimal information and can handle a large number of choice alternatives. In addition, it is called the Hierarchical Bayes model because there are two levels. At the higher level, it is assumed that individuals' preferences are distributed. At the lower level, it is assumed that probability of person  $n$ 's observed choices is governed by a multinomial logit model. These differences enable the HB-ML model to deal with stable individual level results when respondents provide multiple

observations. The mixed logit model had been a relatively unexplored area in the engineering design community until it was recently applied by Wang et al. [23], Shiao et al. [24], Foster et al. [25], Hoyle et al. [26], and Michalek et al. [1].

## 2.2 Noncompensatory choice models

Researchers in economics and psychology have demonstrated that consumers use various heuristics to simplify their choice decisions, whereas the compensatory models use simple additive part-worth rule [27]. By adopting heuristics, a two-stage decision process referred to as a Consider-Then-Choose model has received attention because of its added realism. In a two-stage process, consumers first employ noncompensatory screening rules to narrow their decisions to a small set of products called a consideration set. Then, they use a compensatory choice rule to evaluate products and select one within the consideration set.

This consideration set is determined by consumer's decision-making processes that evaluate product attributes but not product utilities. Thus, noncompensatory choices typically depend on consumers' heuristic methods. Various heuristic decision rules for noncompensatory choices have been proposed in the literature, including conjunctive, disjunctive, lexicographic-by-aspects, elimination-by-aspects, and disjunctions of conjunctions [27]. This article focuses on consider-then-choose models with the conjunctive screening rule that is regarded as the simplest, but still highly effective, heuristic rule.

In the conjunctive screening rule, consumers consider if the product has all "must have" and has no "must not have" aspects. It is formed by multiplying an indicator function across the attribute of an alternative [28]:

$$\prod_m I(x_{jm} > \gamma_m) = 1 \quad (3)$$

where  $x_{jm}$  is the level of attribute  $m$  for choice alternative  $j$ . The cutoff value,  $\gamma_m$ , indicates the smallest level of the attribute that needs to be present for the consumer to consider the alternative [29]. If the alternative has a lower level of the attribute than the cutoff value of a consumer, then the product is screened out. If the level of an attribute is higher than the cutoff value, the attribute is not used in screening.

Advances in Bayesian inference, machine-learning, and greedoid languages make it possible to quantify the consider-then-choose scenarios for a variety of heuristics. Noncompensatory models of the conjunctive screening rules have further developed structure based on Hierarchical Bayes Multinomial Probit (HB-MNP) models. The most significant difference from the HB-MNP is to additionally estimate the cutoff values in the upper level of the hierarchy which can be expressed as [8,28]:

$$\Pr(j)_{nk} = \Pr(U_{nkj} > U_{nki} \text{ for all } i \text{ such that } \prod_m I(x_{nkim} > \gamma_{nm}) = 1) \quad (4)$$

where  $x_{nkim}$  is the level of the attribute for respondent  $n$  in choice scenario  $k$  for alternative  $i$  and attribute  $m$ .  $\gamma_{nm}$  is a respondent-level threshold of attribute  $m$  for respondent  $n$ .

When an attribute is continuously distributed, it is assumed that the cutoff values are normally distributed. When an attribute consists of discrete levels, a multinomial distribution can be adopted such that  $\gamma_{nm} \sim \text{Multinomial}(\theta_m)$ , where  $\theta_m$  is the vector of multinomial probabilities associated with the grid for attribute  $m$ . With the multinomial distribution of cutoffs, each level is tested to determine the highest possible cutoff value ( $\gamma_{nm}^*$ ) from allowable cutoff values ( $\gamma_{nm}^a$ ) using Metropolis-Hastings algorithm [12] based on a probability described as [28]:

$$\gamma_{nm} = \gamma_{nm}^a \text{ with probability } \frac{I(\gamma_{nm}^a)\theta_m}{\sum_l I(\gamma_{nm}^a)\theta_m}, \quad (5)$$

where  $l$  indicates attribute levels. Hence, this model returns an individual's part-worths, cutoff values, and cutoff probabilities at each draw. Disjunctive and elimination-by-aspects rules can also be modeled using Bayesian inference [8,28].

The choice probabilities can be expressed as  $(J-1)$ -dimensional integrals over the differences between the errors because probit models are not closed form [12]. These differences are defines as  $\tilde{U}_{nji} = U_{nj} - U_{ni}$ ,  $\tilde{V}_{nji} = V_{nj} - V_{ni}$ , and  $\tilde{\varepsilon}_{nji} = \varepsilon_{nj} - \varepsilon_{ni}$ . Then, for the consider-then-choose process using probit models the choice probability that individual  $n$  chooses any alternative  $j$  which is in the consideration set can be written as [28]:

$$P_{nj} = \begin{cases} \int I(\tilde{V}_{nji} + \tilde{\varepsilon}_{nji} < 0 \quad \forall j \neq i) \phi(\tilde{\varepsilon}_{ni}) d\tilde{\varepsilon}_{ni} & j \in C_n \\ 0 & j \notin C_n \end{cases} \quad (6)$$

where  $I(\cdot)$  is an indicator of whether the statement in parentheses holds,  $\phi(\varepsilon_n)$  is the joint normal density with zero mean and covariance  $\Omega$ , and  $C_n$  denotes a consideration set of a consumer  $n$ .

## 3. MOTIVATION AND TECHNICAL APPROACH

### 3.1 Motivation

The performance of noncompensatory models has been proven in terms of model fitness and predictability by several other research projects [8,28–32]. It has been shown that noncompensatory models can slightly outperform compensatory models in both hit rate for holdout tasks and likelihood function values. Despite the increased development in noncompensatory choice models, however, compensatory models have been predominantly used in market-based product design. For this reason, this article explores the practicability of using noncompensatory models and compares the results of a product optimization when using compensatory and noncompensatory models in the presence of noncompensatory choices.

Above all, limitations of the noncompensatory models are discussed to determine their practicability in optimization problems. From the standpoint of design optimization, noncompensatory models have some inherent limitations:

- (1) Limitation I: Inadequate screening rule assumptions may

lead to an incorrect estimation of noncompensatory choices because the existing noncompensatory models are derived with specific screening rules. Thus, there is no general form to describe all noncompensatory heuristics. For example, the HB-MNP model with the conjunctive screening rules explained in Section 2.2 can only describe choices that screen out attribute levels lower than the minimum requirements. This disadvantage causes a significant limitation in the model that may be inappropriate for non-incremental levels such as colors.

(2) Limitation II: Aggregate part-worths have an inappropriate form that is difficult to use in optimization problems due to their probabilistic cutoff values. The noncompensatory model results in a set of part-worths and cutoff values at each draw. Since they are interconnected, aggregate part-worths and their corresponding cutoff probabilities have to be integrated into optimization problems at the same time. Although a straightforward approach that uses all draws obtained using MCMC can be considered for optimization problems to present probabilistic cutoff values, this would demand extreme computational power.

(3) Limitation III: Discontinuous choice probability functions (Eq. (6)) associated with noncompensatory models can cause numerical difficulty when precisely solving design constraints during optimization using Genetic Algorithms (GA). This was also pointed out by Morrow et al., who applied nonlinear programming relaxations to noncompensatory models to resolve the discontinuity [9]. As the degree of the variables increase, the numerical difficulty becomes a more problematic issue.

Considering these limitations, compensatory models may be a more suitable form for optimization problems because they have valuable advantages: 1) compensatory models have generalized forms not affected by heuristic rules, 2) aggregate part-worths can be integrated into optimization problems, and 3) likelihood functions are described as continuous functions using logistic or normal distributions. Additionally, if part-worths are extreme in compensatory models, the additive part-worth rule can act like a noncompensatory rule [27]. Thus, noncompensatory choices could be inferred from extreme part-worths in compensatory models.

### 3.2 Technical approach

The rest of this paper explores how compensatory models approximate the two-stage choice process, and what differences are seen in an optimal design search between compensatory and noncompensatory models. The basic idea of behind using compensatory modeling of noncompensatory choices is that strict preferences will be identified for specific attributes, and thus result in distinct population segments. Thus, it is hypothesized that distinct segments would be captured in latent class analysis and extreme part-worth values would be the result of individual-level estimates of the HB-ML model. To verify this hypothesis using the individual-level part-worths, a two-stage process using the LC-MNL and HB-ML models is proposed in this study.

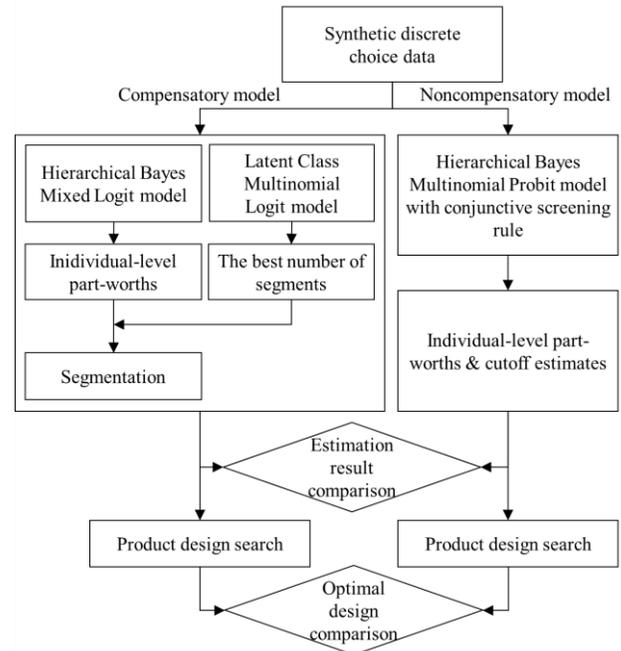


Figure 1. Flowchart of the presented study

Figure 1 displays a flowchart of this study. Synthetic discrete choice data is generated to mimic the consider-then-choose process. The discrete choice data is mathematically modeled using both noncompensatory and compensatory models. The HB-MNP model with a conjunctive screening rule is used as the noncompensatory model. The results of the cutoff distributions and posterior estimates show how the noncompensatory model describes the consider-then-choose process.

In the compensatory model, the LC-MNL and HB-ML models are used. As explained in Section 2, the LC-MNL model is typically used as a tool for market segmentation and the HB-ML model has the ability to deal with continuous representations of heterogeneous preferences. Thus, a latent class analysis is first conducted to segment the population, and then the individual-level preferences are further explored based on the segmentation information. The latent class results are investigated to determine how the noncompensatory attributes result in segmentation in terms of membership probability and the attribute importance. Then, the obtained segments are evaluated using individual part-worths in terms of attribute importance and heterogeneity representation.

Using the synthetic data, a product design problem is solved using Genetic Algorithms (GA) for both the compensatory and the noncompensatory models. So that individual-level preferences can be used in the optimization problem, the HB-ML and HB-MNP models are used as the compensatory and noncompensatory models, respectively. The design results are compared to investigate the suitability of compensatory modeling of the two-stage choice process for product search problems.

## 4. CASE STUDY USING SYNTHETIC CHOICE DATA

Choice data itself does not explicitly state respondents' choice processes. It is impossible to know who really performed

noncompensatory choices and which noncompensatory heuristics were used. Therefore, to analyze the model results, it is important to know the true information about the choice processes of the respondent. In this section, virtual respondents are generated and synthetic choice data is collected based on the heterogeneous preferences of the respondents. Then, part-worth estimates of compensatory and noncompensatory models are obtained and compared. In addition, product design optimization is performed to compare the differences between the two models in optimization problems.

#### 4.1 Generating synthetic choice data

To generate synthetic survey data, a virtual survey is designed around a vehicle selection scenario. Attributes and levels used in this study are described in Table 1. The manual transmission, automatic transmission without shift, and automatic transmission with shift are called MT, AT1, and AT2, respectively. TM is an abbreviation of “transmission” and the capital letter A with a number stands for an attribute. Survey questions are generated using Sawtooth SSI Web [33]. Respondents are asked to evaluate 16 buying scenarios. Each scenario contains four product alternatives and a fifth no-buy option. The survey design is imported into Matlab to conduct choices by virtual respondents.

**Table 1. Vehicle attributes and levels used**

| Level | TM           | Sun-roof | A3 | A4 | A5 | A6 | Price (\$) |
|-------|--------------|----------|----|----|----|----|------------|
| 1     | MT           | No       | 1  | 1  | 1  | 1  | 21,000     |
| 2     | AT w/o shift | Yes      | 2  | 2  | 2  | 2  | 20,000     |
| 3     | AT w/ shift  |          |    | 3  | 3  | 3  | 19,000     |
| 4     |              |          |    | 4  | 4  | 4  | 18,000     |

**Table 2. Pre-defined preferences of virtual respondents**

| Group | number of respondents | Screen out                        | Must-have feature in consideration set |
|-------|-----------------------|-----------------------------------|--|
| 1     | 40                    | MT & No Sunroof                   | AT & Sunroof                           |
| 2     | 40                    | MT                                | AT                                     |
| 3     | 40                    | MT & AT1                          | AT2                                    |
| 4     | 40                    | No Sunroof                        | Sunroof                                |
| 5     | 40                    | only perform compensatory choices |  |

A total of 200 virtual respondents are generated to conduct the consider-then-choose process. Table 2 shows the pre-defined preferences of the respondents used to form consideration sets at the first choice stage. Respondents focus on only transmission and sunroof in noncompensatory choices. Also, lower levels are screened out to follow the definition of the conjunctive screening rule. For example, respondents cannot screen out AT1 only because the conjunctive rule assumes there is a minimum requirement value as a criterion. If a respondent screens out AT1 only, the noncompensatory model with conjunctive rule cannot catch the behavior and considers the respondent to conduct compensatory choices.

The respondents in the pre-defined groups 1 through 4 exhibit noncompensatory behavior to narrow their choice alternatives into a consideration set in the first choice stage. Then, they compare all alternatives in the set and choose one. To mimic a real choice situation, if no alternative in the consideration set satisfies the minimum utility requirement, the no-buy option is selected in the second stage. The respondents in group 5 perform only compensatory choices. To mimic the additive part-worth rule in compensatory choices and introduce heterogeneity, respondents’ preferences (excluding for price) are generated based on uniform distributions with pre-defined intervals. Price preferences are manually generated to allow respondents to prefer lower prices. The virtual survey results in 3,200 observations for model estimation.

**Table 3. Attribute importance of the synthetic data**

| Group | TM   | Sunroof | A3  | A4   | A5   | A6   | Price |
|-------|------|---------|-----|------|------|------|-------|
| 1     |      | 50.0    | 4.2 | 9.6  | 10.1 | 9.6  | 16.6  |
| 2     | 50.0 | 4.7     | 5.0 | 8.2  | 8.8  | 8.5  | 14.9  |
| 3     | 50.0 | 5.7     | 4.2 | 8.6  | 9.6  | 7.5  | 14.4  |
| 4     | 7.1  | 50.0    | 4.8 | 8.0  | 8.3  | 7.8  | 14.0  |
| 5     | 11.7 | 8.1     | 8.7 | 14.9 | 15.1 | 16.1 | 25.3  |

Table 3 shows the attribute importance of the synthetic data for each group. Since attribute importance is calculated based on an additive rule assumption, the importance of the noncompensatory attributes, in fact, cannot be evaluated. However, maximum and minimum utility values of the noncompensatory variables exist which guarantee noncompensatory choices. The utility values are defined as:

$$\begin{aligned} \min(V_{nc}) + \sum_h \max(V_{c,h}) &< V_{no-buy} \\ \max(V_{nc}) + \sum_h \min(V_{c,h}) &> V_{no-buy} \end{aligned} \quad (7)$$

where  $V$  is a part-worth set for each attribute,  $h$  indicates the number of the compensatory attributes, and  $nc$  and  $c$  indicate noncompensatory and compensatory attributes, respectively.  $\max(V_{nc})$  indicates a part-worth of the noncompensatory attribute in the consideration set and  $\min(V_{nc})$  indicates a part-worth set of the noncompensatory attribute excluded from the consideration set. From Eq. (7), the smallest range of  $\max(V_{nc}) - \min(V_{nc})$  is obtained as  $\sum_h \{\max(V_{c,h}) - \min(V_{c,h})\}$ . Hence, the minimum attribute importance of noncompensatory attributes is described as

$$\frac{\max(V_{nc}) - \min(V_{nc})}{\max(V_{nc}) - \min(V_{nc}) + \sum_h \{\max(V_{c,h}) - \min(V_{c,h})\}} \quad (8)$$

Thus, the minimum attribute importance of the noncompensatory attributes is 50%. The importance value is used to check how well compensatory models describe noncompensatory choices in Section 4.2 and 4.3.

**Table 4. Threshold estimates for the posterior means of the conjunctive model**

| Attribute | Level<br>(recorded<br>value) | Possible<br>cutoff | Probability of<br>each cutoff |          | Attribute | Level<br>(recorded<br>value) | Possible<br>cutoff | Probability of<br>each cutoff |          |
|-----------|------------------------------|--------------------|-------------------------------|----------|-----------|------------------------------|--------------------|-------------------------------|----------|
|           |                              |                    | Pre-<br>defined               | Obtained |           |                              |                    | Pre-<br>defined               | Obtained |
| TM        | MT (0)                       | -0.5               | 40 %                          | 38.0 %   | A5        | 1 (0)                        | -0.5               | 100 %                         | 88.6 %   |
|           | AT1 (1)                      | 0.5                | 40 %                          | 38.8 %   |           | 2 (1)                        | 0.5                | 0 %                           | 3.6 %    |
|           | AT2 (2)                      | 1.5                | 20 %                          | 20.6 %   |           | 3 (2)                        | 1.5                | 0 %                           | 2.6 %    |
|           |                              | 2.5                | 0 %                           | 2.7 %    |           | 4 (3)                        | 2.5                | 0 %                           | 2.6 %    |
| Sunroof   | No (0)                       | -0.5               | 60 %                          | 58.1 %   | A6        | 1 (0)                        | -0.5               | 100 %                         | 88.5 %   |
|           |                              | 0.5                | 40 %                          | 39.2 %   |           | 2 (1)                        | 0.5                | 0 %                           | 3.6 %    |
|           | Yes (1)                      | 1.5                | 0 %                           | 2.8 %    |           | 3 (2)                        | 1.5                | 0 %                           | 2.6 %    |
|           |                              | 2.5                | 0 %                           | 2.7 %    |           | 4 (3)                        | 2.5                | 0 %                           | 2.6 %    |
| A3        | 1 (0)                        | -0.5               | 100 %                         | 94.5 %   | Price     | \$21k (0)                    | -0.5               | 100 %                         | 87.6 %   |
|           | 2 (1)                        | 0.5                | 0 %                           | 2.8 %    |           | \$20k (1)                    | 0.5                | 0 %                           | 4.3 %    |
|           |                              | 1.5                | 0 %                           | 2.7 %    |           | \$19k (2)                    | 1.5                | 0 %                           | 2.9 %    |
| A4        | 1 (0)                        | -0.5               | 100 %                         | 89.2 %   | \$18k (3) | 2.5                          | 0 %                | 2.6 %                         |          |
|           | 2 (1)                        | 0.5                | 0 %                           | 3.0 %    | 3.5       | 0 %                          | 2.6 %              |                               |          |
|           |                              | 3 (2)              | 1.5                           | 0 %      |           | 2.6 %                        |                    |                               |          |
|           | 4 (3)                        | 2.5                | 0 %                           | 2.6 %    |           |                              |                    |                               |          |
|           | 3.5                          | 0 %                | 2.6 %                         |          |           |                              |                    |                               |          |

#### 4.2 Noncompensatory choice modeling

Once the virtual survey results were collected, the HB-MNP with conjunctive rule was fit using the R language [34]. Inference was conducted using Bayesian MCMC methods as explained in Section 2.2. The chain was run for the first 5,000 iterations, with the final 5,000 iterations used to estimate the moments of the posterior distributions.

Table 4 shows the aggregate estimates of the probability of cutoff obtained using the conjunctive model. For the discrete attributes, the cutoffs are reported in terms of multinomial point mass probabilities. Each level is recorded as an integer (e.g., 0, 1, 2) in the R code and the recorded values indicate  $x_{nkim}$  in Eq. (4). Thus, a grid of possible cutoff values are also specified (e.g., -0.5, 0.5, 1.5, 2.5), where the lowest cutoff value indicates that all levels are acceptable and respondents performed compensatory choice only, and the highest level indicate that none of the levels are acceptable [28]. A cutoff value of 0.5 indicates that the lowest level is unacceptable and 1.5 indicates the level 1 and 2 (recorded values 0 and 1) are screened out. For example, the 0.5 cutoff value in the transmission attribute indicates the manual transmission is screened out.

The probabilities of each cutoff obtained from the conjunctive model closely correspond with the pre-defined noncompensatory preferences in Table 2. For example, 40% of the total respondent were pre-defined to screen MT out in the virtual survey, and the conjunctive model results in 38% doing so. Estimates of cutoff probability equal to approximately 3% reflect the influence of the prior distribution and the inherent noise of MCMC, and thus can be disregarded for practical purposes. Therefore, it can be concluded that most of the respondents evaluated the attributes 3, 4, 5, and price only in compensatory choices. However, these small percentages do result in small effects on posterior estimates. This is discussed below.

The noncompensatory model fit also returns part-worth estimates at each iteration. The aggregate part-worth estimates of 5,000 draws are listed in Table 5. The analysis of the aggregate part-worths of the transmission and sunroof attributes lead to an interesting insight. Since 60% and 40% of the total respondents perform noncompensatory choices on transmission and sunroof respectively, it should be expected that part-worth estimates of a compensatory model would diverge from zero and show distinct differences between levels.

**Table 5. Estimates for the noncompensatory model**

| Attribute | Level | Posterior<br>mean | Attribute | Level     | Posterior<br>mean |
|-----------|-------|-------------------|-----------|-----------|-------------------|
| TM        | AT1   | 0.09              | A5        | 2         | -0.30             |
|           | AT2   | 0.05              |           | 3         | -0.26             |
|           |       |                   |           | 4         | 0.16              |
| Sunroof   | Yes   | -0.07             | A6        | 2         | 0.07              |
| A3        | 2     | 0.40              |           | 3         | -0.41             |
|           |       |                   |           | 4         | -0.23             |
| A4        | 2     | 0.34              | Price     | \$ 20,000 | 1.78              |
|           | 3     | 0.32              |           | \$ 19,000 | 3.31              |
|           | 4     | 0.15              |           | \$ 18,000 | 5.08              |

However, the conjunctive model estimates are much smaller for these attributes, with the average coefficient approaching zero. This is because the choice probability is evaluated using only the alternatives in the consideration set as shown in Eq. (6). If an alternative is in the consideration set, then its choice probability is determined relative to the other alternatives in the set. If an alternative does not pass the screening rule, then its choice probability is zero. Thus, the screened alternative is not included in the posterior estimation

processes. This leads to competition of alternatives in the consideration set that only has acceptable product features. Eventually, distinctions in posterior estimates between features are diminished and the estimates approach zero. The interaction between posterior estimates and cutoff probabilities allow no exceptions for the attributes that were never screened out by respondents. Hence, the small cutoff probabilities (e.g. 2.6%) also lead to slight degradation of posterior estimates due to the inherent noise of the Bayesian inference technique.

### 4.3 Compensatory choice modeling

#### 4.3.1 Latent class analysis

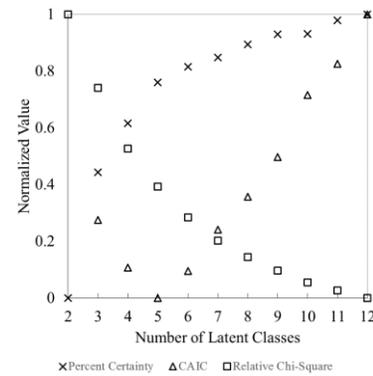
A major focus of this study is to explore how noncompensatory choices are represented in compensatory models. First, how the attributes used to form consideration sets are represented in the compensatory models must be explored. This task is conducted using latent class analysis as a segmentation method because the respondents within each group have relatively similar preferences. However, the preferences are quite different from group to group. Based on the characteristics of latent class analysis, it is hypothesized that if there are distinct attributes which form consideration sets across all individuals, the attributes would play the most significant role in defining different preferences from group to group.

Latent class estimation was conducted using Sawtooth Software’s CBC Latent Class module [35] that estimates a maximum likelihood solution. Figure 2 displays a graphical representation of the normalized statistical measures to assess the goodness of fit: Percent Certainty, Consistent Akaike Information Criterion (CAIC), and Relative Chi-Square [36]. These statistics provide conflicting information on the optimal number of classes in the model. There is neither a single statistic nor a particular combination of these statistics that guarantees the identification of the appropriate number of classes by examining trends across all of the available statistics. Relative Chi-Squared (larger is better) statistics indicate that the 2-class model is the best fit. A contrary trend is seen in the Percent Certainty (larger is better) statistic, which suggests that the 2-class model provides the worst fit. It is noticeable that the two statistics show diminishing decreases for larger numbers of classes. For the CAIC (smaller is better) statistic, there is a certain segment that has the smallest value. Hence, based on the consistency of the trends observed in the statistics, and on the low rate of change in these statistics for models with 5 classes or more, the latent class model fit with 5 classes was selected for use in this work.

**Table 6. Number of members in each group**

|                   |     | Latent class |    |     |    |    | sum |
|-------------------|-----|--------------|----|-----|----|----|-----|
|                   |     | I            | II | III | IV | V  |     |
| Pre-defined group | 1   | 40           |    |     |    |    | 40  |
|                   | 2   |              | 40 |     |    |    | 40  |
|                   | 3   |              |    | 40  |    |    | 40  |
|                   | 4   | 2            |    |     | 38 |    | 40  |
|                   | 5   | 1            | 4  |     | 10 | 25 | 40  |
|                   | sum | 43           | 44 | 40  | 48 | 25 | 200 |

Table 6 shows an interesting comparison result between the predefined group and latent class estimation result. As listed in Table 2, the 200 respondents were divided into five groups expressed using Arabic numbers, according to their preferences in noncompensatory choices. The groups obtained from latent class analysis are expressed using Roman numbers almost correspond to the pre-defined groups. In particular, all respondents in the groups 1, 2, and 3 remain in the segments I, II, and III in latent class estimation, respectively. Also, it is interesting that the 40 respondents in group 4, who were defined to screen the no-sunroof feature, are divided into segments I and IV. Since the ‘no sunroof’ feature is screened out of both segments I and IV, the 2 respondents moved from group 4 to segment I are still considered to maintain their preferences. The estimates of the respondents in group 5 depend on random preference generation and survey design. Even though the respondents did not conduct noncompensatory choices, if cumulative choices are rationally biased, it could be defined as a member of the segment that does make noncompensatory choices. This is because the latent class analysis does not estimate noncompensatory choices, but simply groups respondents with similar preferences.



**Figure 2. Normalized statistics for LC-MNL model**

**Table 7. Membership probability of belonging to a group**

| Total number of respondents | Latent class |       |       |       |       |
|-----------------------------|--------------|-------|-------|-------|-------|
|                             | I            | II    | III   | IV    | V     |
| 43                          | 99.88        | 0     | 0     | 0.12  | 0     |
| 44                          | 0.99         | 98.68 | 0     | 0     | 0.33  |
| 40                          | 0            | 0.01  | 99.99 | 0     | 0     |
| 48                          | 0            | 0     | 0     | 99.96 | 0.04  |
| 25                          | 0            | 1.67  | 0     | 0.01  | 98.31 |

While the statistics used to display Fig. 2 allow researchers to determine the best number of segments by comparing relative goodness of fit, membership probability shows how well respondents are categorized into the groups. Latent class estimation assumes that each respondent is not wholly in one group but considered to have some non-zero probability of belonging to each group. This probability is called membership probability. If the solution fits the data very well, the membership probabilities approaches one for the group the respondent is in and zero for the other groups. As shown in Table 7, respondents are assigned dominant probabilities of

memberships in only one class. The average maximum membership probability of the 5 classes is 99.36%. This means that the latent class model results in a very good estimation and the respondents are assigned to each group with high certainty.

The attributes used to form consideration sets may be inferred from attribute importance. Table 8 implies which attributes play a significant role in each group. The noncompensatory attributes of each segment are valued at above 50% importance. This suggests that noncompensatory choices can be maintained because the minimum importance for the noncompensatory choice is obtained as 50% in the previous section. In addition, it is observable that the attributes that result in the formation of consideration sets have much higher importance than the other attributes. For example, the importance value of transmission in the segment III is 82.9, while the importance of the other attributes are all much lower. Thus, it can be supposed that the transmission is the most significant feature in segment III, and the other attributes are trivial compared to the transmission. In contrast, in segment V, dominant attributes except the price do not exist because the segment implies compensatory choices. The results of importance indicate that the discrete classes are determined from the features used in noncompensatory choices.

**Table 8. Attribute importance of latent class analysis**

| Latent class | TM   | Sunroof | A3  | A4  | A5   | A6  | Price |
|--------------|------|---------|-----|-----|------|-----|-------|
| I            | 70.5 |         | 3.5 | 4.6 | 3.1  | 5.3 | 13.0  |
| II           | 56.4 | 2.0     | 1.1 | 4.0 | 4.0  | 2.8 | 29.7  |
| III          | 82.9 | 0.3     | 1.0 | 3.0 | 1.7  | 2.0 | 9.1   |
| IV           | 3.4  | 50.7    | 2.6 | 5.1 | 4.9  | 2.5 | 30.8  |
| V            | 5.7  | 11.6    | 5.1 | 9.9 | 10.4 | 9.1 | 48.2  |

Despite the strong inference about the attributes used in noncompensatory choice, the latent class analysis does not explicitly tell if it is likely that a noncompensatory screening rule can be used, but just gives a presumption that there were possibly strong preferences in certain attributes. Notice that the result in Tables 7 and 8 show that if there are noncompensatory choices and the latent class results in estimates with high certainty, attribute importance would be higher than 50% at the segment-level. However, it doesn't mean that if an attribute importance is above 50%, noncompensatory choices would be conducted for that segment.

#### 4.3.2 Hierarchical Bayes mixed logit model

While the number of segments and the features forming consideration sets are speculated using the LC-MNL model, individual-level preferences are estimated using the HB-ML model. This section provides details on how the HB-ML model represents the two-stage choices by investigating part-worths, attribute importance, and rank orders of the feature levels.

Table 9 reports aggregate part-worth estimates for the HB-ML model. The HB-ML model was fit using the Sawtooth Software CBC/HB module [37] that was set to perform 10,000 random draws for each respondent before assuming convergence and averaging the next 10,000 random draws to minimize the error. It is observable that the transmission and sunroof attributes have more deviated posterior means because

they are used to form consideration sets in noncompensatory choices. The extreme part-worth allows the additive rule used in compensatory choices to mimic a noncompensatory screening rule. In contrast, the posterior means of the other attributes are relatively flat.

**Table 9. Part-worth estimates for the HB-ML model**

| Attribute | Level | Posterior mean | Attribute | Level     | Posterior mean |
|-----------|-------|----------------|-----------|-----------|----------------|
| TM        | AT1   | 6.92           | A5        | 2         | -0.15          |
|           | AT2   | 9.89           |           | 3         | -1.36          |
|           |       |                |           | 4         | 0.08           |
| Sunroof   | Yes   | 5.61           | A6        | 2         | 0.79           |
| A3        | 2     | -0.37          |           | 3         | 0.18           |
|           |       |                |           | 4         | 0.42           |
| A4        | 2     | 0.17           | Price     | \$ 20,000 | 3.39           |
|           | 3     | -0.34          |           | \$ 19,000 | 3.53           |
|           | 4     | 0.23           |           | \$ 18,000 | 6.47           |

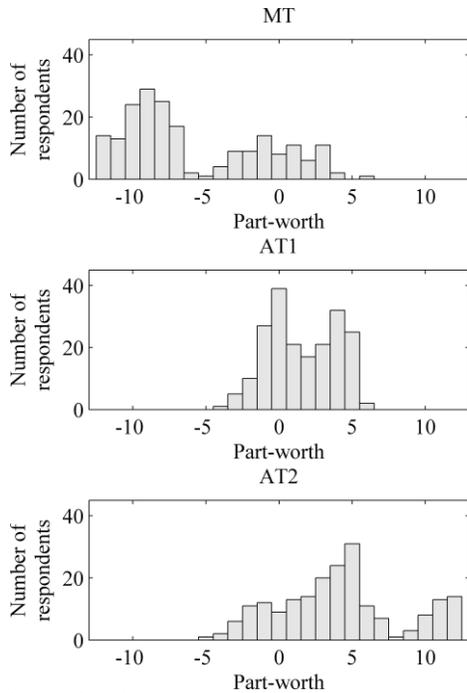
**Table 10. Zero-centered part-worth of each segment**

| Latent class | Transmission |      |      | Sunroof |      |
|--------------|--------------|------|------|---------|------|
|              | MT           | AT1  | AT2  | No      | Yes  |
| I            | -8.4         | 3.1  | 5.3  | -6.7    | 6.7  |
| II           | -7.6         | 3.6  | 4.0  | 0.5     | -0.5 |
| III          | -10.2        | -0.7 | 10.9 | -0.3    | 0.3  |
| IV           | -0.1         | -0.2 | 0.3  | 6.9     | -6.9 |
| V            | -0.4         | 0.0  | 0.4  | 0.2     | -0.2 |

Table 10 shows the aggregate part-worth estimates of each segment. The segment information is borrowed from the latent class analysis, and the individual-level estimates obtained using HB-ML model are classified according to the segment. Only two attributes used in noncompensatory choices are listed with zero-center. To reconfirm the existence of the distinct segments corresponding to the latent class analysis, the individual-level estimates are classified according to the segment information of the latent class analysis. Thus, the second hypothesis, that extreme part-worth values would be caused in individual-level estimates, is verified using the segment-level part-worths and the distributions of the individual-level part-worths.

Table 10 shows that the segments I through IV have extreme part-worths in their attributes while the part-worths of segment V are relatively flat. Notice that the attributes used to form consideration sets have extreme values, while the other attributes have relatively flat part-worths. The extreme part-worths at the individual-level are also observed in the histogram displayed in Fig. 3.

The existence of the extreme part-worths is significant to the probability calculation of the additive rule. If a part-worth is large enough, it could act like a noncompensatory attribute in the additive rule. In other words, although the compensatory model doesn't explicitly describe the noncompensatory choices resulting in binary preferences (Must-have or Screen-out) for noncompensatory attributes, the great part-worths can result in extreme preference, e.g. 99% or 1%., for the noncompensatory attribute in the additive rule.



**Figure 3. Histogram of aggregate posteriors for transmission attribute**

In the aggregate estimates of the HB-ML model, it is also noticeable that the AT1 feature of the segment III is relatively flat but MT has an extreme value, despite the fact that the two features were screened out at the same time in the virtual survey. This is not a special case in the commercial software used; the same tendency was observed in the HB-ML estimates obtained using Matlab code. It is guessed that the flat part-worth is caused by the prior distribution assumption. Details are discussed below.

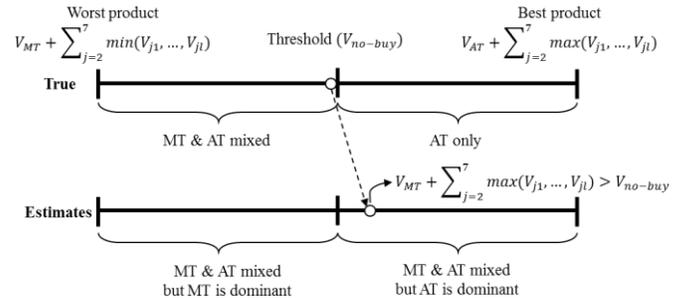
Figure 3 displays histograms of individual-level aggregate posteriors for the transmission attribute. MT and AT2 features result in more extreme part-worth values than AT1. In addition, all three histograms are closer to multimodal distributions than normal distributions. These results lead to an interesting discussion about the hierarchical Bayesian model of noncompensatory choices. The distribution of heterogeneity has to be specified in hierarchical Bayesian models and the multivariate normal distribution is most commonly used. The commercial software used in this study also adopts the multivariate normal distribution. However, when the true distribution of heterogeneity is as close to a finite mixture of normal distributions as the noncompensatory choices, it is inappropriate to use a multivariate normal assumption for the distribution of heterogeneity. Thus, a hierarchical Bayes model might over-estimate the proportion of the part-worths near zero [38]. The over-estimation near zero was observed in the results of both software packages. Hence, exploring appropriate prior distribution assumptions or incorporating prior knowledge would be required for the HB-ML model of noncompensatory choices, and is a source of future work.

Attribute importance values listed in Table 11 are obtained using the individual-level part-worths obtained from the HB-ML fit. The results indirectly show how well the HB-ML model

describes the pre-defined preferences. As discussed in Section 4.1, the noncompensatory variables have to have at least 50% importance; group 1 in the HB-ML model satisfies this condition. However, notice that groups 2, 3, and 4 contain importance values lower than 50%. This implies that the HB-ML model does not completely approximate the noncompensatory choices. This outcome conceptually corresponds with the implication of the extreme part-worth values, that the HB-ML does not explicitly describe the noncompensatory choices.

**Table 11. Attribute importance of HB-ML model**

| Pre-defined group | TM   | Sunroof | A3  | A4   | A5   | A6   | Price |
|-------------------|------|---------|-----|------|------|------|-------|
| 1                 |      | 67.9    | 2.8 | 3.7  | 6.1  | 4.9  | 14.5  |
| 2                 | 41.2 | 7.1     | 6.0 | 7.2  | 8.8  | 9.0  | 20.6  |
| 3                 | 43.7 | 8.3     | 5.2 | 8.2  | 9.6  | 8.6  | 16.4  |
| 4                 | 11.5 | 42.2    | 5.6 | 6.5  | 7.3  | 6.9  | 20.0  |
| 5                 | 14.8 | 11.8    | 8.0 | 10.7 | 13.6 | 13.6 | 27.6  |



**Figure 4. Conceptual diagram of compensatory modeling of noncompensatory choice**

The presence of importance values below 50% means that there is some switching of products across the threshold of selection. Figure 4 describes the switching of a product having an MT feature and the largest part-worths of each compensatory attribute for a respondent who screened out MT in choice tasks. Even though the respondent never selected the MT feature in the virtual survey, some products having MT can be selected in the market simulation using the part-worths of the HB-ML model. This is due to the absence of a strict threshold to form consideration sets in compensatory models. Nevertheless, since product search problems only focus on the several top products, the misrepresentation may be ignored in optimization problems. Regarding this argument, details follow below.

The results in Table 12 help explain why the absence of a strict threshold may have only an insignificant role in optimization problems. Among 160 respondents from groups 1 to 4 who performed noncompensatory choices, 150 respondents can reproduce the pre-defined screening rules. Only 10 respondents show violations of noncompensatory choices using the HB-ML estimates. In addition, the utility gaps between the best product and the switched product are significantly large. Their ignorable odd ratio values also show the violated products could not have a significant effect on an optimum product search.

**Table 12. Part-worth comparison of the switched products**

| Pre-defined group | No. of respondents having switched products | Avg. utility of the best product* | Avg. utility of the best product among switched products** | Threshold ( $V_{no-buy}$ ) | Odd Ratio*** |
|-------------------|---|-----------------------------------|--|----------------------------|--------------|
| 2                 | 4   | 16.3                              | 1.5  | 1.0                        | $\cong 0$    |
| 3                 | 2   | 20.7                              | 8.7  | 8.0                        | $\cong 0$    |
| 4                 | 4   | 22.5                              | 4.0  | 1.8                        | $\cong 0$    |

$$* V_{nc,consideration} + \sum \max(V_{c1}, \dots, V_{cl})$$

$$** V_{nc,screen\ out} + \sum \max(V_{c1}, \dots, V_{cl})$$

$$*** \frac{\exp(V_{nc,screen\ out} + \sum \max(V_{c1}, \dots, V_{cl}))}{\exp(V_{nc,consideration} + \sum \max(V_{c1}, \dots, V_{cl}))}$$

As a statistic to demonstrate the resemblance between the true preference and HB-ML fit, Spearman's rank correlation coefficient of the attribute importance is calculated. The two data sets are compared for 768 possible product feature combinations for each respondent, which results in 30,720 observations for each group. Higher correlation coefficient values imply a better estimation ability of the HB-ML model to describe the true preference.

**Table 13. Spearman's rank correlation coefficient**

| Pre-defined group | Spearman's rank correlation coefficient |
|-------------------|---|
| 1                 | 0.711                                   |
| 2                 | 0.712                                   |
| 3                 | 0.629                                   |
| 4                 | 0.695                                   |
| 5                 | 0.641                                   |

The results of the correlation coefficient give rise to an interesting discussion about the implications of choice between the compensatory and noncompensatory models. As seen in Table 13, groups 1, 2, and 4 result in higher coefficient values than group 5, who performed compensatory choices only. This result suggests that noncompensatory choices could be more accurately estimated than compensatory choices by the compensatory model. However, group 3 has the lowest correlation coefficient among the five groups. This could be affected by the prior normal distribution assumption.

#### 4.4 Product design search

This section focuses on differences between the compensatory and noncompensatory models in design problems when searching for optimal product configurations. In particular, take note of the differences in the consideration sets drawn from the two models. It is a decisive factor to compare the two models because one of the most significant challenges in developing a new product is to ensure that the new product will be considered by consumers.

The individual-level aggregate estimates of the HB-ML model are used as the compensatory model. In contrast, as

explained in Section 3.1, it is difficult to use the aggregate values of the conjunctive model in optimization problems due to the probabilistic representation of cutoffs and corresponding part-worth estimates. For this reason, 5000 draws generated in the MCMC process have to be used in the optimization problem for accuracy, even though this makes solving optimization problems computationally expensive. In this study, every 10th draw was kept for optimization to reduce the computational effort. Thus, 500 draws were saved per individual.

**Table 14. Pricing structure**

| Level | TM   | Sunroof | A3  | A4  | A5  | A6  |
|-------|------|---------|-----|-----|-----|-----|
| 1     | 0    | 0       | 0   | 0   | 0   | 0   |
| 2     | 800  | 500     | 500 | 100 | 200 | 100 |
| 3     | 1000 |         |     | 200 | 300 | 200 |
| 4     |      |         |     | 300 | 400 | 300 |

Pricing structure information for each product attribute is shown in Table 14. In addition to this pricing structure, the base price of \$18,000 is added. To calculate the part-worths for the price attribute, a piecewise linear interpolation is assumed.

**Table 15. Attribute levels of competitor products in the market**

| Competitor | TM  | Sunroof | A3 | A4 | A5 | A6 | Price(\$) |
|------------|-----|---------|----|----|----|----|-----------|
| Product 1  | MT  | No      | 1  | 1  | 1  | 1  | 18,000    |
| Product 2  | AT1 | Yes     | 1  | 2  | 3  | 3  | 19,900    |
| Product 3  | AT2 | Yes     | 2  | 4  | 4  | 4  | 21,000    |

The objective of optimization problems is to maximize the choice probability of the optimized product configuration in a competitive market. The competitor products are defined as shown in Table 15. Competitor product 1 is the cheapest, while product 3 is the most expensive product in the market. The objective is to maximize the share of preference that is based on the choice probability calculated using Eqs. (2) and (6) for the compensatory and noncompensatory models, respectively.

The first scenario is to find the best product, maximizing choice probability among the 768 possible product configurations in the market. This scenario does not necessarily require search algorithms because the best product can be easily found when evaluating the 768 products using all draws in market simulation. The second scenario is to find the optimal configurations of two products that maximize the choice probability among the 589,056 product combinations. For the second scenario, a Monte-Carlo simulation is impractical due to the computational burden. One market simulation using the noncompensatory model takes approximately 15 seconds on a laptop computer running an Intel i7 2.20 GHz with 16GB RAM.

Thus, it is roughly calculated that it would take about 102 days to complete 589,056 market simulations. Hence, a Genetic Algorithm (GA) based approach was used for both problems because a GA works directly with the discontinuous choice probabilities and the existing studies show the feasibility of a GA in a product line search [22,25]. When using a GA for the second scenario, computational time was reduced to approximately three days when the pool size was 200 and the stall generation limit was 50.

**Table 16. Optimal product configuration (Scenario 1)**

| Model           | TM  | Sunroof | A3 | A4 | A5 | A6 | Price (\$) | $SOP_s(\%)$ | $SOP_t(\%)$ | design error (%) |
|-----------------|-----|---------|----|----|----|----|------------|-------------|-------------|------------------|
| True            | AT2 | Yes     | 1  | 3  | 1  | 2  | 20,100     | 37.1        | 37.1        | -                |
| Compensatory    | AT2 | Yes     | 1  | 2  | 1  | 2  | 20,300     | 52.4        | 35.9        | 3.2              |
| Noncompensatory | AT2 | Yes     | 1  | 1  | 3  | 3  | 20,350     | 41.3        | 29.2        | 21.3             |

**Table 17. Optimal product configuration (Scenario 2)**

| Model           | TM  | Sunroof | A3 | A4 | A5 | A6 | Price (\$) | $SOP_s(\%)$ | $SOP_t(\%)$ | design error (%) |
|-----------------|-----|---------|----|----|----|----|------------|-------------|-------------|------------------|
| True            | AT1 | Yes     | 1  | 1  | 3  | 3  | 19,850     | 54.2        | 54.2        | -                |
|                 | AT2 | Yes     | 1  | 2  | 1  | 2  | 20,300     |             |             |                  |
| Compensatory    | AT1 | Yes     | 2  | 2  | 1  | 3  | 19,900     | 68.7        | 47.8        | 11.8             |
|                 | AT2 | Yes     | 1  | 2  | 1  | 2  | 20,300     |             |             |                  |
| Noncompensatory | AT2 | Yes     | 2  | 1  | 3  | 3  | 20,550     | 61.8        | 45.6        | 15.9             |
|                 | AT2 | Yes     | 1  | 4  | 2  | 4  | 20,400     |             |             |                  |

Tables 16 and 17 show the optimization solutions of the two scenarios, respectively. The approach for investigating the differences of the two models in a product design search is divided into two aspects - must-have features and design error. The ability to capture the predefined must-have features is the most important statistic on which to evaluate the effectiveness of the model for product design. Design error can be quantified by evaluating the optimum design of the estimated preference using the true preference.  $SOP_s$  indicates the share of preference of the optimum design evaluated using the part-worths used in the design search problem.  $SOP_t$  indicates the share of preference of the optimum design evaluated using the true preference. The design error metric is therefore  $(SOP_t - SOP_s)/SOP_t$ .

In the design solution of a one-product problem, it is noticeable that the three data sets result in the same product features for the noncompensatory attributes (transmission and sunroof) that were used to form consideration sets. In addition, the compensatory model results in five features identical to the true values, and four identical to the noncompensatory model. These results imply that if there are strong noncompensatory choices, the noncompensatory attributes can be found, whatever models are chosen. Hence, the must-have features can be included in the product configuration regardless of model selection. In an aspect of design error, it is interesting that the compensatory model results in significantly better performance than the noncompensatory model. This suggests the optimum design from the compensatory model would work better in the market than the design from the noncompensatory model.

Similar trends are observed in the result of scenario 2 shown in Table 17. The true data and the compensatory model resulted in the same transmission and sunroof features, though there is discrepancy in some of the compensatory attributes. Although the noncompensatory model resulted in only AT2 for the transmission feature, the crucial finding is that both models are able to lead a new product line that would be included in consideration sets. In an aspect of design error, it is also interesting that the compensatory model results in better performance than the noncompensatory model.

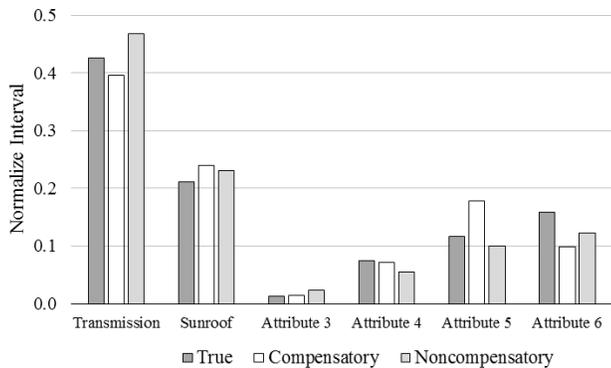
**Table 18. Choice probability interval**

| Data            | Interval between max and min choice probabilities (%) |         |     |     |      |     |
|-----------------|---|---------|-----|-----|------|-----|
|                 | TM  | Sunroof | A3  | A4  | A5   | A6  |
| True            | 19.2  | 9.5     | 0.6 | 3.4 | 5.3  | 7.2 |
| Compensatory    | 36.6  | 22.2    | 1.4 | 6.6 | 16.4 | 9.1 |
| Noncompensatory | 36.7  | 18.2    | 1.8 | 4.3 | 7.9  | 9.7 |

For compensatory attributes 3 to 6, there exists both commonality and discrepancy in the optimal product configurations. To investigate how many differences exist between the three solutions, choice probabilities are tested at all attribute levels. Table 18 shows choice probability intervals when only one attribute changes its level while the other attributes are fixed as their optimum designs. Then, the results will show how sensitive the share of preference is to each attribute for each model.

First, the interval of the compensatory and noncompensatory models has a larger magnitude than that of the true data. This implies that the two models exaggerate preferences and make the differences of products more distinct. Second, the noncompensatory attributes have greater sensitivity than the compensatory attributes. This empirical sensitivity analysis strongly confirms the hypothesis that noncompensatory attributes play a larger role than the compensatory attributes in the search for an optimum design.

Figure 5 displays normalized probability intervals for each model. This figure shows the attribute importance in the product search problem. The transmission attribute that leads the noncompensatory choices at 60% results in the largest probability interval. Also, the sunroof attribute that leads noncompensatory choices at 40% results in the second largest probability interval. In contrast with the transmission and sunroof, the compensatory attributes have smaller intervals than the two noncompensatory attributes. This means that the compensatory attributes are relatively insignificant in product search problems if there are strong noncompensatory attributes.



**Figure 5. Interval between the max. and min. choice probabilities of each attribute**

The compensatory and noncompensatory models result in very similar solutions in product design search problems. In particular, it is noticeable that the product features used to form consideration sets are commonly obtained in both optimal designs. Also, even though there is a discrepancy between solutions, it is a relatively insignificant factor in market simulation in terms of choice probability. In this study, the compensatory model can be more accessible for product design optimization because there is no concern about the limitations of noncompensatory models, such as computational burden or an incorrect screening rule assumption. To support the finding and the argument in this paper, further investigation of the suitability of the compensatory model for the two-stage choices is needed considering various choice scenarios in terms of model accuracy and predictive ability at both the individual and population levels.

## 5. CONCLUSIONS

The main purpose of this paper is to explore the suitability of compensatory models to mimic the consider-then-choose process for a product design search. This is motivated by the limitations of the existing Bayesian-based noncompensatory model, which can be summarized by potential errors in screening rule assumptions, probabilistic representations of noncompensatory choices, and discontinuous choice probability functions. It is hypothesized that distinct segments would be captured in latent class analysis and extreme part-worth values would be the result in individual-level estimates of the HB-ML model.

To verify this hypothesis, this research first investigates segmentation techniques of the two-stage choice process using latent class analysis. Using latent class is based on the idea that noncompensatory choices would cause a distinct differentiation of population preference. The numerical results of latent class analysis confirm this hypothesis. The distribution of preference heterogeneity is explored to compare the true preference and the compensatory model at individual-level preference. The results of the individual-level preference analysis show that the HB-ML model can represent noncompensatory choices using extreme values in part-worths despite the absence of strict thresholds. Lastly, implications of model choice between the two representations of the consider-then-choose process are discussed using the results of the product design search

problem. The results of the product design search show interesting implications of model form choice. Although there are several insignificant differences between the two models in the market simulation, the compensatory model would be preferred for noncompensatory choices due to smaller design errors and its relatively inexpensive computational burden in product design optimizations.

A limitation of this work is that the attributes used in noncompensatory choices may not have the largest importance in the latent class model when only a small quantity of the respondents conducted noncompensatory choices. However, this should not be a concern in a view of the product design search because optimization problems intend to draw solutions maximizing or minimizing an objective across all respondents. In this case, the small number of the respondents performing noncompensatory choices would not have a significant effect on the optimization problem.

Future work from this research will focus on three different aspects: 1) developing optimization techniques using the compensatory model of the two-stage choice process, 2) resolving the disadvantage arising from a normal distribution assumption of priors, 3) exploring differences between compensatory models of several noncompensatory heuristics, and 4) investigating the implications of the discrepancy between the true preference distribution and the prior distribution assumption of Bayesian inference for product design.

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