Proceedings of the ASME 2016 International Design Engineering Technical Conferences &
Computers and Information in Engineering Conference
IDETC/CIE 2016
August 21−24, 2016, Charlotte, North Carolina, USA

DETC2016-59535

PRODUCT LINE DESIGN SEARCH CONSIDERING RELIABILITY AND ROBUSTNESS UNDER UNCERTAINTY IN DISCRETE CHOICE METHODS

Jaekwan Shin
Graduate Research Assistant
North Carolina State University
Raleigh, NC 27695, USA
jshin5@ncsu.edu

Scott Ferguson[1]
Associate Professor
North Carolina State University
Raleigh, NC 27695, USA
scott_ferguson@ncsu.edu

ABSTRACT
Point-estimates of part-worth values in customer preference models have been used in market-based product design under the simplifying assumption that customer preferences can be treated as deterministic. However, customer preferences are not only inherently stochastic, but are also statistical estimates that exhibit random errors in model formulation and estimation. Ignoring uncertainty in customer preferences and variability in estimates has caused concern about the reliability and robustness of an optimal product design solution. This study quantitatively defines reliability and robustness of a product design under uncertainty when using discrete choice methods. These metrics are then integrated into a multi-objective optimization problem to search for product line solutions considering reliability and robustness under uncertainty when using discrete choice methods.

Keywords: market-based product design; product line; design optimization; discrete choice model; reliability; robustness

1. INTRODUCTION
According to [1], "*human decision-making involves trading off costs or benefits, which are known now with certainty, with risky outcomes that will occur in the future*." "*From a social science perspective, these decisions are associated with varying levels of probability (risk) and uncertainty because of missing information (ambiguity)* [2]." When using discrete choice models to estimate customer preference, certainty and uncertainty in choice behavior has been discussed using observable and unobservable parameters.

Uncertainty in customer preference estimates associated with discrete choice methods is a matter of individual-level decision behavior. Mixed logit models have typically been used to estimate preference heterogeneity, allowing variation in taste across individuals. Parameters of random utility models consist of a vector of preference coefficients (observable) and a random error term (unobservable). The observable vector of the preference coefficients is specified to be a multivariate normal distribution. The error term reflects specification errors, omitted factors, non-observable factors, and unobserved heterogeneity of preferences [1]. If the error term is independent and identically distributed (i.i.d.) with a Type I extreme value distribution, and the maximum utility rule is applied in simulations, the expectation is identical to the logit model [3]. The hierarchical Bayes mixed logit (HB-ML) model assumes preference heterogeneity as a continuous distribution, and Bayesian inference is employed to estimate posterior distributions of the preference coefficients. Using numerical integration in estimation requires many draws of posterior distributions to be generated. Rather than using the whole posterior distribution, point-estimates are found by taking the mean value of the posterior distribution and these values are used in a market simulation because of their reduced computational cost.

While many who use market simulators to make decisions commonly use point-estimates of individual's part-worths, this ignores uncertainties that are inherent to discrete choice methods. Customer preferences are not only inherently stochastic, but also are statistical estimates that exhibit errors in model form and estimation procedures. For this reason, there are concerns about the reliability and robustness of an optimal design solution under the presence of uncertainty when using discrete choice methods.

Figure 1 illustrates how variations in part-worth estimates can influence the objective function in market-based design problems. If a single-objective problem is posed where the goal is to maximize market share of the product line and point-estimates of preference are used, as shown in Fig. 1(a), the best

[1] Corresponding Author

design is solution A. However, when variability within the preference estimates is considered, solution A is no longer the clear winner. Solutions B or C may capture more market share than solution A, as visualized by the box plots. In multi-objective formulations, variation in the part-worth estimates leads to objective function values better described as surfaces, as shown in Fig. 1(b). Hence, choosing the optimal design is more difficult when preference estimate variations are considered.

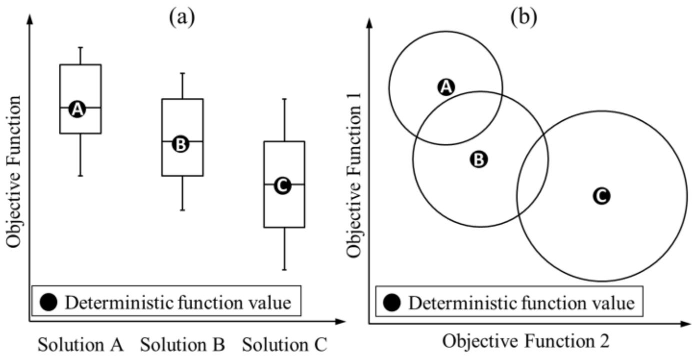

Figure 1. Variability in hypothetical solutions [4]**:**
(a) Single-objective (b) Multi-objective

The objective of this paper is to propose an optimization problem to search for product line solutions considering reliability and robustness under uncertainty when using discrete choice methods. When uncertainty in discrete choice methods is considered, selecting a final solution is made more difficult by the variability that occurs in objective function values. Therefore, additional criteria are required to evaluate the tradeoffs between design solutions when uncertainty sources are considered. With this purpose, realiability and robustness of a product line design are characterized. Draws from a Bayesian-based mixed logit model are used with a Randomized First Choice (RFC) simulation to investigate how part-worth variation impacts the optimal design solution. A multi-objective optimization problem is then developed that incorporates reliability and robustness in product line design optimization. Section 2 provides background knowledge about discrete choice models and the quantification of uncertainty in market-based product design. Section 3 introduces definitions of reliability and robustness. Section 4 presents a numerical study to investigate how demand uncertainty impacts the results of a product line search, and a multi-objective problem formulation is proposed. Conclusions, limitations, future work are discussed in Section 5.

2. BACKGROUND

The necessary background knowledge regarding discrete choice models is introduced to aid in the explanation of this study. Section 2.1 briefly introduces the fundamental concepts of discrete choice models capable of estimating individual-level part-worths – hierarchical Bayes mixed logit (HB-ML). Section 2.2 presents HB draws and RFC as methods to account for variation in the demand model.

2.1 Discrete Choice Models

Discrete choice analysis is used to model product demand by capturing a customer's choice behavior [5]. The choice utility that person n obtains from alternative i can be expressed as a sum of an observed utility V_{ni} and an unobserved random disturbance ε_{ni} as in Eq. (1) [3,6]:

$$U_{ni} = V_{ni} + \varepsilon_{ni} = \beta_n^T x_{ni} + \varepsilon_{ni} \quad (1)$$

Here, β_{ni} is a vector of part-worths for the individual and x_{ni} is a vector of values describing the configuration of design alternative i. Usually, β_n is unknown and is estimated statistically. When the unobserved random disturbance, ε_{ni}, is independent and identically distributed (i.i.d.) and represented by a Gumbel distribution [3], the difference between two extreme value distributions has a logistic distribution. Using the logit model, the choice probability that person n chooses an alternative i is obtained from the simple logit formula shown in Eq. (2) [3]:

$$P_{ni} = \frac{e^{\beta_n^T x_{ni}}}{\sum_j e^{\beta_n^T x_{nj}}} . \quad (2)$$

Preference heterogeneity is defined as a variation in taste across individuals [7] and leads to differentiated product specifications. The most widely used model to represent heterogeneity is the HB-ML model. The HB-ML model defines individual-level preferences using continuous distribution functions. By setting model parameters as multivariate normal distributions, the model can estimate individual-level part-worths using Bayesian inference and Markov-Chain Monte-Carlo (MCMC) methods [3]. It is called a hierarchical model because there are two levels. The assumption at the higher level is that an individual's preferences are normally distributed. At the lower level, a multinomial logit model is assumed to quantify the choice probability [8].

2.2 Uncertainty in Discrete Choice Methods

In engineering, ambiguity and vagueness of sustem variables or parameters are considered as primary sources of uncertainties [9]. Ambiguity is generally due to noncognitive (aleatory) sources that include: inherent physical randomness, statistical uncertainty, and modeling uncertainty [9]. Thus, it is irreducible uncertainty. Vagueness is due to cognitive (epistemic) sources such as limited knowledge and human factors. Thus, these are reducible [10].

In discrete choice methods, aleatory uncertainty can be caused by dynamics on demand and cost [11], inherent preference inconsistency [12], and response variability [13,14]. Epistemic uncertainty is caused by choice context [15,16], sampling errors in Bayesian inference [17,4,18], and demand model misspecification [19–21].

To quantify the reliability and robustness of a design under uncertainty in demand modeling, it is necessary to address stochastic preference coefficients and directly use their probability distributions in a design problem. However, by taking point-estimates of part-worths, certain uncertainty effects are ignored in a market simulation. In this study, two methods are investigated to account for the uncertainty in discrete choice methods: HB draws and RFC.

2.2.1. HB draws

Estimating an HB model requires a number of iterations before convergence is assumed. The hierarchical Bayesian framework, which is implemented using MCMC techniques, yields complete posterior distributions of the preference coefficients at the individual-level. Thus, multiple estimates, called draws, are generated that form the posterior distributions of each respondent's preference coefficients. These draws are then averaged for each respondent to create a single vector of part-worths that represent preferences for each attribute level included in the study. Then, the single value is used as a best guess of random parameters ignoring variability in parameters. In market-based product design, the point-estimates of part-worth coefficients have been typically used as a simplifying assumption to reduce the computational burden of market simulations. To represent the variability associated with the Bayesian procedure, the draws themselves can be used in market simulations instead of using point-estimates [4,22,23].

2.2.2. Randomized First Choice (RFC)

A first choice model assumes respondents choose the product alternative with the highest utility value from a competitive set (maximum utility rule) [24]. RFC modifies this process by introducing error terms into the utility equation during the simulation phase. Multiple part-worth values can be created by adding random errors to the aggregate part-worths obtained from the HB model. Then, a choice simulation is conducted using these modified part-worths following the maximum utility rule. RFC was first introduced as a simulation technique by Orme and Huber [25,26] in response to product similarity challenges. They demonstrated that using this formulation in a market simulator outperformed four commonly used models in predicting holdout choice shares: an aggregate multinomial logit model, a latent class model, an individual choice analysis of the latent class, and a HB-ML model. However, RFC can become computationally demanding, making it challenging to use in large-scale market-based design problems.

RFC adds two kinds of variability to the individual-level part-worths. The utility of alternative i for an individual n is derived as [25]

$$U_{ni} = (\beta_n + E_{a,n})^T x_{ni} + E_{p,n} \tag{3}$$

Here, E_a is a vector of variability added to the part-worths and E_p is a variability added to product i. Intuitively, attribute variability represents inconsistency in a respondent's relative weights or part-worths applied to product attributes [25]. The attribute variability term reflects variation in taste [27,28]. Product variability occurs when a customer evaluates choice alternatives inconsistently in several different choice tasks [25]. In logit models, product variability is mathematically equivalent to the unobserved random disturbance in Eq. (1). One of Gumbel distribution and normal distribution can be selected for E_a, but a Gumbel distribution has to be used to define E_P for logit models [22]. Hence, a tuning process is required to determine the degree of variability. After introducing the variabilities to the point-estimates of part-worths, the first choice rule is simulated to predict choice shares.

While preference share methods are tunable for scale and usually more precise than a first choice simulation, they suffer from IIA (independence from irrelevant alternatives) issues [24]. First Choice Share (FCS) can resolve IIA issues but usually results in biased predictions and is not tunable for scale [24]. The RFC model combines the desirable aspects of first choice and share of preference choice rules by introducing variations in point-estimates and simulating choices using the maximum utility rule many times. RFC simulations can resolve the extreme choice share issue by tuning the extent of attribute and product variation in Eq. (3).

3. METHODOLOGY

This paper introduces a quantitative measurement of reliability and robustness of a product line design in market-based design using RFC simulation. A multi-objective problem formulation is proposed to integrate the stochastic aspects into one framework. Reliability and robustness are quantitatively defined in Sec. 3.1 and 3.2, respectively. Section 3.3 describes a multi-objective problem formulation used to search for a non-dominated set of product line solutions under uncertainty in discrete choice models.

3.1 Reliability of Market-based Product Design

In engineering terminology, reliability is defined as "*the probability of successful performance*"; thus it is the converse of the term probability of failure [10]. In this study, reliability indicates satisfactory performance of a product line design. Reliability-Based Design Optimization (RBDO) [29] is "*a method to achieve confidence in product reliability at a given probabilistic level* [30]." Various methods have been developed to advance reliability analysis and design methods: sampling-based design using MCS (Monte-Carlo Simulation) [31], MPP (Most Probable Point) [32] based double-loop RBDO using FORM (First-Order Reliability Method) [33,34] and SORM (Second-Order Reliability Method) [35], and a single-loop method called SORA (Sequential Optimization and Reliability Assessment) [36]. This study adopts sampling-based reliability analysis using MCS because no analytical method has been developed to define uncertainty in discrete choice models and market simulations.

Using the notion of failure to characterize unwanted behavior, two different types of "simulation failures" are proposed when designing a product line using discrete choice methods and individual-level choice behavior:

Failure-I: *A respondent fails to choose one of the products associated with the product line design solution in a single RFC replication.*

Figure 2 illustrates Failure-I. Offerings in a product line, which are determined from the solution to an optimization problem, are called *Own Offerings* or *Own Products* in the rest of the paper. There are three *Own Offerings*, three competitor products, and a no-buy option in this simulated market. The numbers 1 and 0 represent a respondent's product selection in a first choice simulation. The bit string $\mathbf{c}$ expresses the first choice result using RFC replicates. For example, if a consumer chooses *Own Product* 1 (P1) in the first choice simulation using RFC replicates, the choice result is saved as $\mathbf{c} = [1000000]$.

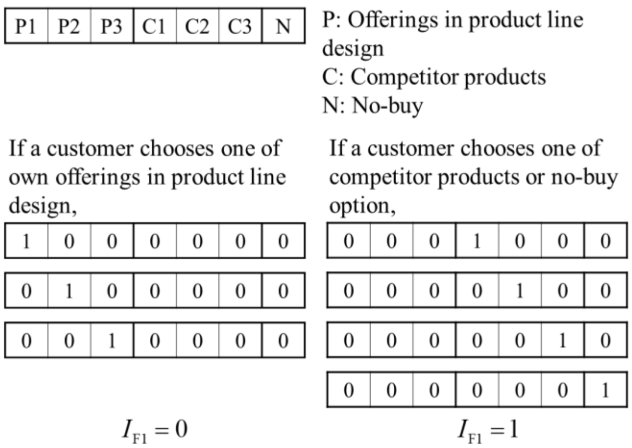

Figure 2.Visualization of Failure-I for a Single RFC Replicate

$I_{F1}(\mathbf{c})$ is an indicator function to count Failure-I and is defined as

$$I_{F1}(\mathbf{c}) \equiv \begin{cases} 0, & \mathbf{c} \in \Omega_{PL} \\ 1, & \text{otherwise} \end{cases}. \tag{4}$$

Ω_{PL} is a product line set that consists of *Own Offerings*. If one of *Own Offerings* is not selected in the first choice simulation using RFC replicates, $I_{F1}(\mathbf{c}) = 1$. This means the product line design fails under uncertainty when using discrete choice methods.

The probability of Failure-I is defined as

$$P_{F1} \equiv P\left[\mathbf{c} \notin \Omega_{PL}\right] = \frac{1}{N}\frac{1}{R}\sum_{n=1}^{N}\sum_{r=1}^{R} I_{F1}^{n,r}(\mathbf{c}_{n,r}). \tag{5}$$

N and R indicate the number of respondents and RFC replicates, respectively. Reliability-I, defined as $1 - P_{F1}$, represents how much market share is expected under demand uncertainty when introducing the product line solution into the market. Thus, Reliability-I represents the first choice share of a product line using RFC replicates.

Failure-II: *A respondent changes their product choice decision (made using deterministic preference coefficients) to a different product or 'none' within the choice alternatives when the respondent is given an identical choice again.*

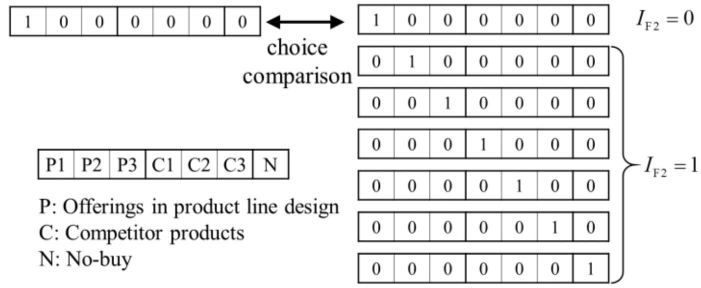

Figure 3. Visualization of Failure-II

Figure 3 illustrates Failure-II. $I_{F2}(\mathbf{c})$ is an indicator function to count Failure-II and defined as

$$I_{F2}(\mathbf{c}) \equiv \begin{cases} 0, & \mathbf{c} \in \Omega_{D} \\ 1, & \text{otherwise} \end{cases}. \tag{6}$$

Ω_D is a deterministic choice set obtained using point-estimates. If a respondent changes their choice decision made using aggregate part-worths to a different product in RFC simulation, $I_{F2}(\mathbf{c}) = 1$. The probability of Failure-II is defined as

$$P_{F2} \equiv P\left[\mathbf{c} \notin \Omega_{D}\right] = \frac{1}{N}\frac{1}{R}\sum_{n=1}^{N}\sum_{r=1}^{R} I_{F2}^{n,r}(\mathbf{c}_{n,r}). \tag{7}$$

Reliability-II, defined as $1 - P_{F2}$, suggests how effectively the deterministic product line design works in the simulated market when uncertainty is considered when using discrete choice methods.

3.2 Robustness of Market based Product Design

In engineering terminology, robustness is defined as "*the ability to tolerate the effect of uncertainty or variation in design parameters without eliminating the source of the uncertainty or variation* [37,38]." Robust design is a method to improve design quality by minimizing the effect of uncertainty on the output performance function [30]. It can be categorized into two types based on the source of variation [39]:

Type-I: "*minimizing performance variations caused by variations in uncontrollable parameters* [39]*"* as described in Fig. 4.

Type-II: "*minimizing variations in performance caused by variations in design variables* [39]*"*.

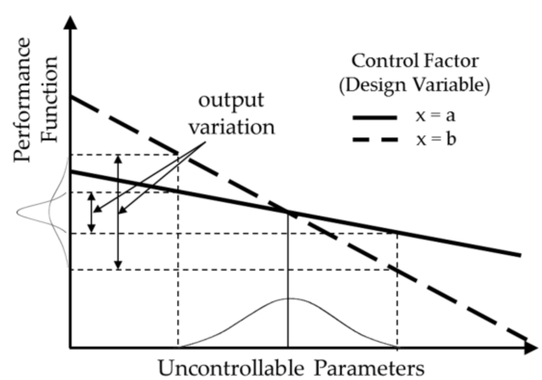

Figure 4. Robust design Type-I [39]

Robustness in market-based product design can be defined as the ability to tolerate the effect of uncertainty in discrete choice methods. In product line optimization problems, design variables are commonly defined as the factors that embody product features. Preference coefficients are used as parameter values to define a market model and are not controlled in the optimization problem. Thus, robust design for market systems belongs to Type-I (if the design variables are not subject to uncertainty) because demand uncertainty is uncontrollable. A robust design problem for market-based product design can therefore be defined by minimizing objective function variability under demand uncertainty. In this study, First Choice Share predictions of a product line design using RFC replicates are chosen as the objective function, and variability is measured using the standard deviation (SD) of FCS values.

3.3 Product Line Design under Uncertainty

A product line search can be developed as a triple-objective optimization problem that contains the reliability and robustness definitions discussed in Sec. 3.1 and 3.2. Thus, the problem aims to maximize an average choice share (Reliability I) and its robustness while minimizing the probability of Failure-II. The formulation of the product line search problem is expressed as:

$$\begin{array}{ll} \text{maximize} & 1\text{-}P_{F1} \\ \text{minimize} & \sigma_{FCS} \\ \text{minimize} & P_{F2} \\ \text{with respect to} & \text{X = product configurations of own offerings in a product line} \\ \text{subject to} & \text{Lower and upper bounds of each attribute} \end{array} \quad (8)$$

Eq. (8) is a simplified representation of product line search problems, and many variants could be further developed. A company seeking an opportunity to adopt a market-based product design strategy would need to reflect limitations and decisions concerned with manufacturing, marketing, or engineering design. Design problems should be able to control any of these limitations and the decisions associated with product feature configuration. The limitations and decisions associated with these other domains could be formulated as design variable constraints.

The triple-objective product line search problem returns a Pareto optimal set and dominated solutions consisting of many different product line configurations. After these results are returned, a process is needed to help the decision maker choose the best design from this set. For example, a formal multi-attribute decision method such as the hypothetical equivalent and inequivalent method (HEIM) [40] could be adopted. However, this aspect of the design process is not presented in this paper and remains as future work.

4. NUMERICAL STUDY

Task procedures of the numerical study are described in Fig. 5. Generating synthetic choice data is presented in Sec. 4.1. The HB-ML model is fit using the synthetic discrete choice data and RFC replicates are drawn in Sec. 4.2. Deterministic design and the analysis of robustness and reliability are conducted in Sec. 4.3. Finally, Sec. 4.4 discusses a multi-objective product line search problem considering reliability and robustness under uncertainty in discrete choice methods.

4.1 Generating Synthetic Choice Data

Simulated discrete choice data is used in this study to ensure answer consistency in the choice task questions and to control the amount of RFC replicates needed. To generate synthetic survey data, a tablet PC selection scenario is introduced. The attributes and levels used in this study are described in Table 1. The capital letter A with a number stands for an attribute. Survey questions are generated using Sawtooth SSI Web [41]. Respondents are asked to evaluate 15 buying scenarios including five hold-out questions. Each scenario contains three product alternatives and a fourth no-buy option.

Synthetic preference data is generated by:

Step 1. Generate deterministic synthetic preferences
- Generate μ_β (mean) for each attribute: $U(-1,1)$
- Generate σ_β (standard deviation) for each attribute: $U(0.5,1)$
- Generate β_n (individual's preference) for each attribute: $N(\mu_\beta,\sigma_\beta)$
- Generate β_{no-buy} (no-buy threshold) for each respondent: $U(0.6, 0.8)$

Step 2. Variation in the deterministic preference
For each choice task of each respondent
- Generate taste variation for each attribute: $U(-0.5, 0.5)$
- Generate variation in the no-buy threshold: $U(-0.1, 0.1)$

Table 1. Tablet PC attributes and levels

	Attribute				
	A1	A2	A3	A4	Price
	Connec-tivity	Processor	Screen Size	Storage	
Level 1	Wi-Fi	Entry	7 inch	16 GB	$ 200
Level 2	Cellular	Mid	8 inch	32 GB	$ 400
Level 3		High-End	10 inch	64 GB	$ 600
Level 4			12 inch	128 GB	$ 800
Level 5				256 GB	

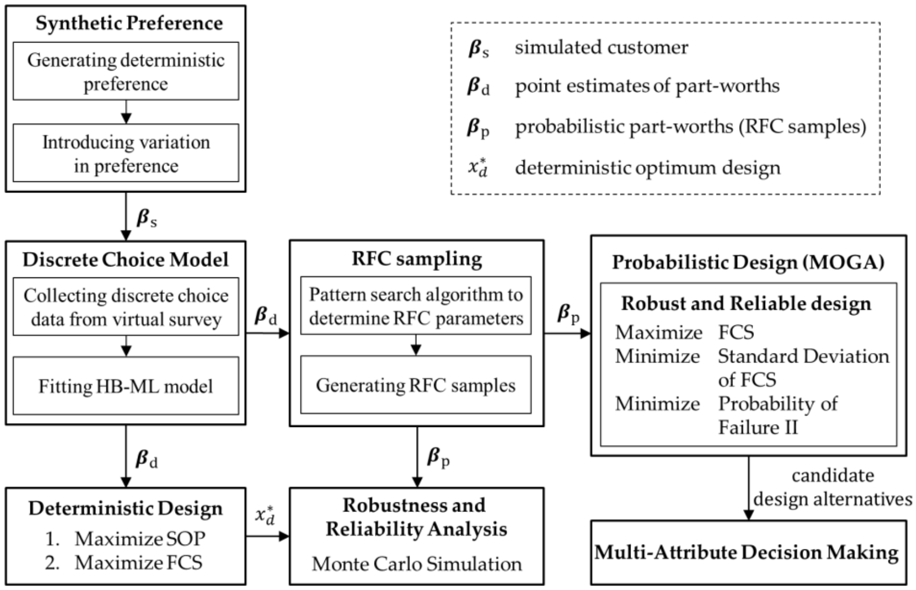

Figure 5. Flowchart of the presented study

4.2 Quantifying Variation in Demand Model

The individual-level part-worths of the HB-ML model are obtained using Sawtooth Software's CBC/HB module [8], and 10,000 burn-in iterations are performed. Then, 10,000 draws after burn-in are saved (skipping every hundredth draw) per respondent, for a total of 2,000,000 (200 respondents×10,000 draws) sets of 19 part-worths.

Determining how many draws of HB and how many sampling replications for RFC to use is significant as it affects the accuracy and efficiency of simulation. An existing study [22], whose survey size is similar to the synthetic data generated in this study, used at least 100 draws per respondent based on a reliability test between replicates. The study compared MAE (Mean Absolute Error) values to its 95th percentile value. 100 replicates per respondent may be acceptable in terms of the internal reliability test, however, 100 replicates would not be enough to guarantee accuracy of the probability of failure when using Monte Carlo Simulations (MCS). In MCS, using 100 samples allows for a 1% probability interval of failure. For a reliable estimate, at least ten times the minimum is usually recommended [10]. In this paper, 10,000 replicates per respondent are used to predict the probability of failure at a level of 0.1%.

Another challenge is determining the RFC parameters that represent variations in the aggregate part-worths. Since there are 14 product attribute levels, there are 14 normal distributions for E_a and a Gumbel distribution for E_p. Their mean values should be exactly the same as the point-estimates. Therefore, 14 standard deviation values must be determined for the normal distribution. In Table 2, the standard deviation of each preference coefficient is represented as $\sigma_{\text{attribute,level}}$. For the Gumbel distribution, the mean is given by $E(X) = \upsilon + \gamma\alpha^{-1}$, where υ is a location parameter, α is a scale parameter, and γ is the Euler-Mascheroni constant that is approximately equal to 0.5772 [42]. By setting $E(X) = 0$ to maintain the point-estimates, the two parameters can be divided into independent and dependent variables. In this study, the scale parameter is searched as $\sigma_P = \alpha^{-1}$ to determine E_p in Eq. (3).

A pattern search algorithm is used to search for the RFC parameters that minimize the mean absolute error (MAE) in predicting choice shares using holdout questions [25]. For example, suppose there are three products and they have 20, 30, and 50 choice shares, respectively. If we obtains the predicted first choice shares as 10, 20, and 70, respectively, the MAE value is calculated as $\left(|20-10|+|30-20|+|50-70|\right)/3 = 13.3$. A smaller MAE for the holdout questions implies better predictive ability. Table 2 shows MAE values of each data. As suggested in Orme and Baker's study [22], using HB draws does not result in a smaller MAE value than RFC data in simulations, despite the simplified assumptions about the attribute and product variation distributions. According to [22], *a reverse number of levels effect, and an excluded level effect*, can explain why RFC is more effective than using HB draws when considering the predictive power of market simulation.

The mechanism of the search process is simple but computationally expensive due to the size of the replicates. In this study, the RFC parameters are obtained using 1,000 replicates (RFC 1k data). Then, 10,000 replicates called A-RFC 10k data are generated using the parameters. The feasibility of the augmented RFC samples in product search problems is investigated. As shown in Table 3, the RFC 1k data results in exactly the same product line design as the RFC 10k data. This result suggests that reducing sample size in a parameter search problem would be acceptable in terms of product search objectives. The augmented RFC data also results in exactly the same product line solution with the RFC 1k data. This implies the A-RFC 10k data can maintain the original variation information of the RFC 1k in the design problem. However, to have confidence in the data augmentation technique for RFC sampling, increased validation is necessary in future work.

Table 2. Comparison of MAE values for each data

Data	N	RFC parameters															MAE	Enhan-cement
		$\sigma_{1,1}$	$\sigma_{1,2}$	$\sigma_{2,1}$	$\sigma_{2,2}$	$\sigma_{2,3}$	$\sigma_{3,1}$	$\sigma_{3,2}$	$\sigma_{3,3}$	$\sigma_{3,4}$	$\sigma_{4,1}$	$\sigma_{4,2}$	$\sigma_{4,3}$	$\sigma_{4,4}$	$\sigma_{4,5}$	σ_P		
Point-estimates	1																9.60	Datum
HB draws	10,000																6.58	31.5%
RFC 1k	1,000	0.12	0	3.04	0	0	0	0.54	0.36	0	0	0.26	0	3.06	3.32	0.16	3.63	62.2%

Table 3. Optimal product line solution of each RFC data

Data	Objective	FCS (%)	Product 1				Product 2				Product 3			
			A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
RFC 1k A-RFC 10k RFC 10k	Maximize FCS	65.23 65.38 65.58	2	2	1	4	2	2	4	3	2	1	3	4

4.3 Single-Objective Product Line Search

This section investigates the effect of uncertainty by quantifying the reliability and robustness of a product line solution obtained using part-worth point-estimates. Then, A-RFC 10k data is used to evaluate the product line design to introduce uncertainty. Reliability and robustness of the deterministic design are quantified using the definitions in Sec. 3.1 and 3.2.

Table 4 shows the pricing structure for each product attribute level. A base price of $200 is added. To calculate the part-worths for the price attribute, a piecewise linear interpolation is assumed. The competitor products in a simulated market are defined as shown in Table 5.

Table 4. Pricing structure

Level	A1	A2	A3	A4
1	0	0	0	0
2	40	80	40	40
3		160	80	80
4			160	120
5				160

Table 5. Attribute levels of competitor products

Competitor	A1	A2	A3	A4	Price
Product 1	2	1	1	5	$ 400
Product 2	1	3	2	3	$ 480
Product 3	2	2	4	4	$ 600

Analysis of a latent class model [43,44] suggests that creating three *Own Offerings* in a product line is the most effective number of products to offer. The solution to the deterministic product line problem is shown in Table 6. Two different objectives are set: maximizing FCS and maximizing SOP. One *Own Product* is identical in the two solutions, but two *Own Products* are different. Notice that *Own Product* 3 of the SOP solution is identical to the third competitor product, because the share of preference simulation cannot resolve the IIA issue.

Table 7 shows the reliability and robustness of the design solutions obtained in Table 3 and Table 6. Each design solution is evaluated using A-RFC 10k replicates. Solution A indicates the optimum design of FCS problem using A-RFC 10k data and is shown in Table 3. Solutions B and C represent the optimum designs of FCS and SOP problems, respectively, which are shown in Table 6.

Solution C has a smaller FCS than Solution B. This is because the *Own Product* 1 of Solution C and competitor product 3 divide FCS equally due to the presence of a duplicate offering in the choice set. The FCS value of Solution B decreases from 78.50% when using point-estimates to 63.46% when using RFC replicates. Since first choice share predictions can resolve IIA issues, first choice share decreases when evaluated using RFC replicates [24].

If a decision-maker is only concerned about average FCS values, the best solution is A because it has the largest choice share. However, when variability in the demand model is considered, Solution A is no longer the clear winner. Figure 6 describes distributed FCS values of the three design alternatives under variation in the demand model. It is obvious that there is a chance that Solutions B or C can capture more market share than Solution A as described in box plots. For the error bars, the central mark indicates the median value, the box indicates the 25th and 75th percentiles, the whiskers extend to approximately $\pm 2.7\sigma$ with normal distribution assumption. Outliers outside $\pm 2.7\sigma$ are individually plotted [45].

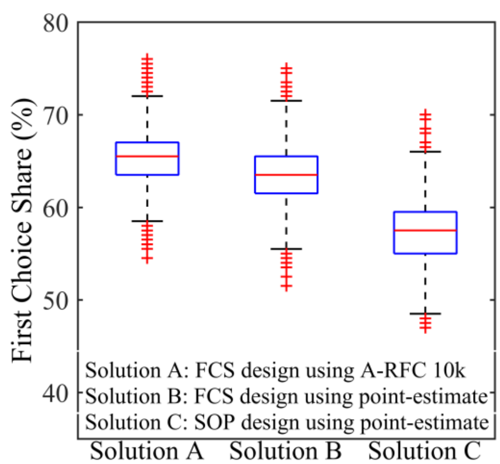

Figure 6. Variability in performance function

Table 6. Deterministic optimal product line solution

Data	Objective	Product 1				Product 2				Product 3				Function value
		A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4	
Point-estimate	Maximize FCS	2	2	4	3	2	2	1	5	2	2	4	2	FCS = 78.50 %
	Maximize SOP	2	2	4	3	2	2	1	2	2	2	4	4	SOP = 77.79 %

Table 7. Robustness and reliability analysis

Solution	Data	Objective	FCS (%)				P_{F2} (%)
			Avg	Min	Max	SD	
A	A-RFC 10k	Maximize FCS	65.38	54.5	76.0	2.69	35.42
B	Point-estimates	Maximize FCS	63.46	51.5	75.0	2.98	38.35
C		Maximize SOP	57.25	47.0	70.0	3.11	32.42

To support design decisions under variation in the demand model, this study adopts the reliability and robustness definitions explained in Sec. 3.1 and 3.2. Results of robustness and reliability analysis for the three alternatives are listed in Table 7. All numbers in Table 7 are evaluated using A-RFC 10k data to quantify reliability and robustness under the uncertainty of A-RFC 10k data.

The standard deviation of FCS values are used as a measure of design robustness, and the probability of Failure-II represents the reliability of a design under demand variation. For these results, there is no clear preferred solution alternative:

- Reliability-I (FCS): A > B > C
- Reliability-II: C > A > B
- Robustness: A > B > C

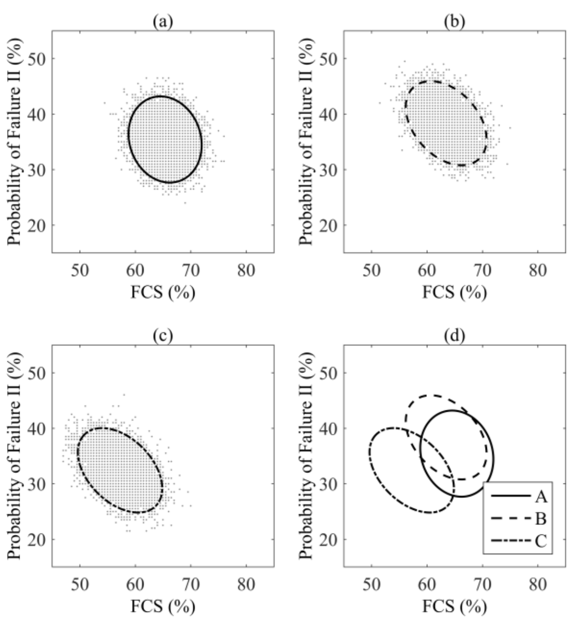

Figure 7. Distributed FCS and probability of Failure-II: (a) Solution A (b) Solution B (c) Solution C (d) 95% confidence ellipse of each solution

Figure 7 7 depicts where solution conflicts occur in terms of reliability and robustness by describing FCS values over the 10,000 RFC replicates, and the corresponding reliability of Failure-II. Each circle indicates a 95% confidence ellipse of all samples that can be drawn from the underlying normal distribution. In Fig. 7-(d), it is difficult to determine a clear preferred solution alternative as there is considerable overlap between Solutions A and B. Also, there is a chance that Solution C may capture more share and have less simulation failure than the other designs. For this reason, considering multiple objectives was proposed in Sec. 3.4 and the results of this analysis are presented in the next section.

4.4 Multi-Objective Product Line Search Considering Reliability and Robustness

To formulate a multi-objective product line search problem under uncertainty, the optimization algorithm is changed to the elitist non-dominated sorting GA (NSGA-II) [46,47] to take an advantage of its sorting algorithm. The size of the population at each generation is 120 because it is ten times the number of design variables. To enhance solution quality, a targeted initial population [48] is generated. Two elite individuals are guaranteed to survive each generation. Binary tournament based on crowding distance is used as a selection operator, while crossover occurs using arithmetic means. The mutation operator is based on the Gaussian distribution. For integer encoding, each component is rounded to the nearest integer. Finally, convergence is met when 50 generations are performed with no improvement in the best fitness function value. When using A-RFC 10k data, one generation took approximately 50 minutes using a desktop running an Intel i7-2600 Quad-Core Processor 3.40 GHz with 8GB RAM. The total run time is approximately 10.3 days with 300 generations.

The triple-objective product line search problem returns a Pareto optimal set and dominated designs consisting of many different product configurations. Figure 8 displays the Pareto optimal set and the dominated designs evaluated in the evolutionary algorithm. There are 37 unique solutions (black dots) in the Pareto optimal set and 6,020 unique designs (red dots) in the dominated set. The solutions obtained using the multi-objective optimization problem represent candidate product line designs. Thus, selecting one design solution from all candidates in Fig. 8 is important to consider trade-offs between reliability and robustness in decision-making process.

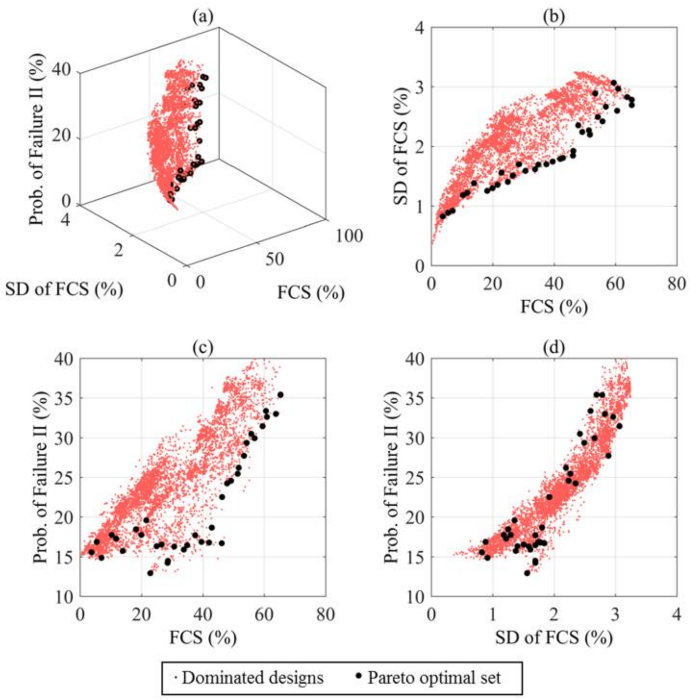

Figure 8. Pareto design alternatives of product line search problem: (a) 3-D (b) FCS vs SD of FCS (c) FCS vs Prob. of Failure II (d) SD of FCS vs Prob. of Failure II

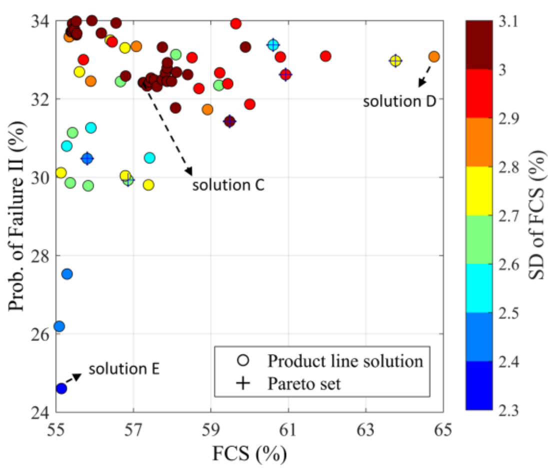

Figure 9. Refined design solutions

To narrow down the number of possible design alternatives, solutions capturing a predicted choice share below ($FCS^{tar} = 55\%$) are eliminated. Also, solutions that do not meet a target reliability level ($P_{F2}^{tar} = 34\%$) are excluded from consideration. After applying these filters, 71 candidates remain, including six non-dominated solutions as shown in Fig. 9. Selecting one solution among the set of alternatives in the feasible range is required but difficult because there are tradeoffs between FCS, SD of FCS, and the probability of Failure-II. For example, selecting one solution among solutions C, D, and E in Fig. 9 is difficult because of tradeoffs between function values as shown in Table 8. Solution C is the optimum design maximizing SOP obtained using point-estimates. Solution D has the largest FCS value among the candidate solutions in Fig. 9. Solution E has the smallest SD of FCS and the probability of Failure-II in Fig. 9. To finalize design decision considering reliability and robustness in a multi-objective framework, a multi-attribute decision method such as HEIM would be needed to ensure that a rational decision is made. This will be applied in future work.

Table 8. Solution comparison

Solution	FCS (%)	SD of FCS (%)	P_{F2} (%)
C	57.25	3.11	32.42
D	64.76	2.83	33.08
E	55.14	2.32	24.60

5. SUMMARY AND DISCUSSION

The main contribution of the study lies in specifying the reliability and robustness of a product design under uncertainty when using discrete choice methods, and integrating these measures into a multi-objective optimization framework. In the proposed approach, an RFC model is used to introduce variation in the point-estimates of an HB-ML model. By simulating many choices, the RFC simulation can resolve issues associated with IIA and extreme choice share. An efficient search procedure is then proposed to quantify the degree of variation. A multi-objective optimization problem formulation is introduced using reliability and robustness to search for a set of non-dominated design alternatives. Then, a multi-attribute decision method could be applied to support making design decisions.

This work demonstrates that a product line decision can be enhanced to exhibit greater realism by considering uncertainty when using discrete choice models. The definition of Reliability-II can improve the choice probability of a firm's own product line design under uncertainty. Also, a product line exhibiting robustness would have the ability to tolerate perturbations. In conclusion, the solution quality associated with the non-dominated set is enhanced by considering uncertainty in discrete choice methods using RFC simulation and multi-objective optimization techniques.

The proposed work has some limitations and future work is recommended. The degree of variation in RFC replicates depends on holdout question design because the parameter search problem in Sec. 4.2 aims to minimize the mean absolute error in predicting holdout choice shares. Thus, methodologies to design holdout questions are necessary to enhance validity of the uncertainty quantification method.

Another potential shortcoming is that the proposed optimization problem would be computationally intensive because of using many RFC replicates, though an efficient parameter search procedure and a targeted initial population for the multi-objective problem are applied. Thus, enhancing computational efficiency would be further developed for practical use. If demand uncertainty and market simulation can be expressed using correlated continuous distributions at the individual-level preference, analytical methods could be further explored to reduce computational burden. As computationally less expensive methods, analytical reliability analysis methods such as FORM and SORM when using a mixed logit model if: 1) demand uncertainty could be expressed using continuous distributions with correlations in preference structure, 2) closed-form choice probabilities are available, and 3) market simulators could handle continuous distributions.

Furthermore, developing tailored GA algorithms to draw designs close to Pareto set as many as possible is recommended to enhance the quality of product design search. Drawing quality designs is significant because a winning design may not be in the Pareto optimal set when considering a decision maker's preference in a multi-attribute decision making method. Supervised genetic algorithms need to be explored to produce elaborated offspring. Lastly, the design alternatives obtained in the multi-objective problem should be evaluated using a multiattribute decision making method to determine a single product offering.

ACKNOWLEDGEMENTS

We gratefully acknowledge support from the National Science Foundation through NSF CAREER Grant No. CMMI-1054208. Any opinions, findings, and conclusions presented in this paper are those of the authors and do not necessarily reflect the views of the National Science Foundation.

REFERENCES

[1] De Palma, A., Ben-Akiva, M., Brownstone, D., Holt, C., Magnac, T., McFadden, D., Moffatt, P., Picard, N., Train, K., Wakker, P., and Walker, J., 2008, "Risk, uncertainty and discrete choice models," Marketing Letters, **19**(3-4), pp. 269–285.

[2] Hsu, M., Bhatt, M., Adolphs, R., Tranel, D., and Camerer, C. F., 2005, "Neural systems responding to degrees of uncertainty in human decision-making," Science, **310**(5754), pp. 1680–1683.

[3] Train, K., 2009, Discrete choice methods with simulation, Cambridge university press.

[4] Von Hagel, K. A., and Ferguson, S. M., 2015, "Simulating Variability of Rework Cost and Market Performance Estimates in Product Redesign," ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (IDETC/CIE2015), Paper No. DETC2015-47598.

[5] Green, P. E., and Srinivasan, V., 1990, "Conjoint Analysis in Marketing: New Developments with Implications for Research and Practice," Journal of Marketing, **54**(4), p. 3.

[6] Chen, W., Hoyle, C., and Wassenaar, H. J., 2013, Decision-Based Design, Springer, London.

[7] Allenby, G., and Rossi, P., 1998, "Marketing models of consumer heterogeneity," Journal of Econometrics, **89**(1-2), pp. 57–78.

[8] Sawtooth Software inc., 2014, "Sawtooth Software CBC/HB 5.5.3."

[9] Ayyub, B. M., and McCuen, R. H., 2011, Probability, statistics, and reliability for engineers and scientists, CRC press.

[10] Haldar, A., and Mahadevan, S., 2000, Probability, reliability and statistical methods in engineering design, John Wiley & Sons, Incorporated.

[11] Raman, K., and Chatterjee, R., 1995, "Optimal Monopolist Pricing Under Demand Uncertainty in Dynamic Markets," Management Science, **41**(1), pp. 144–162.

[12] MacDonald, E. F., Gonzalez, R., and Papalambros, P. Y., 2009, "Preference Inconsistency in Multidisciplinary Design Decision Making," Journal of Mechanical Design, **131**(3), p. 031009.

[13] Louviere, J., 2001, "What If Consumer Experiments Impact Variances as well as Means? Response Variability as a Behavioral Phenomenon," Journal of Consumer Research, **28**(3), pp. 506–511.

[14] Salisbury, L. C., and Feinberg, F. M., 2010, "Alleviating the Constant Stochastic Variance Assumption in Decision Research: Theory, Measurement, and Experimental Test," Marketing Science, **29**(1), pp. 18–22.

[15] Louviere, J., Street, D., Carson, R., Ainslie, A., Deshazo, J. R., Cameron, T., Hensher, D., Kohn, R., and Marley, T., 2002, "Dissecting the Random Component of Utility," Marketing Letters, **13**(3), pp. 177–193.

[16] Swait, J., Adamowicz, W., Hanemann, M., Diederich, A., Krosnick, J., Layton, D., Provencher, W., Schkade, D., and Tourangeau, R., 2002, "Context Dependence and Aggregation in Disaggregate Choice Analysis," Marketing Letters, **13**(3), pp. 195–205.

[17] Wang, X., and Curry, D. J., 2012, "A robust approach to the share-of-choice product design problem," Omega, **40**(6), pp. 818–826.

[18] Besharati, B., Luo, L., Azarm, S., and Kannan, P. K., 2006, "Multi-Objective Single Product Robust Optimization: An Integrated Design and Marketing Approach," Journal of Mechanical Design, **128**(4), pp. 884–892.

[19] Gilbride, T. J., and Lenk, P., 2010, "Posterior predictive model checking: An application to multivariate normal heterogeneity," Journal of Marketing Research, **47**(5), pp. 896–909.

[20] Abramson, C., Andrews, R. L., Currim, I. S., and Jones, M., 2000, "Parameter Bias from Unobserved Effects in the Multinomial Logit Consumer Choice," Journal of Marketing Research, **37**(4), pp. 410–426.

[21] Montgomery, A. L., and Bradlow, E. T., 1999, "Why Analyst Overconfidence About the Functional Form of Demand Models Can Lead to Overpricing," Marketing Science, **18**(4), pp. 569–583.

[22] Orme, B. K., and Baker, G. C., 2000, "Comparing Hierarchical Bayes Draws and Randomized First Choice for Conjoint Simulations," Sawtooth Software Research Paper Series.

[23] Shin, J., and Ferguson, S., 2015, "Modeling Noncompensatory Choices With a Compensatory

Model for a Product Design Search," ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (IDETC/CIE2015), Paper No. DETC2015-47632.
[24] Orme, B. K., 2006, Getting started with conjoint analysis : strategies for product design and pricing research, Research Publishers, LLC, Madison, WI.
[25] Huber, J., Orme, B. K., and Miller, R., 1999, "Dealing with Product Similarity in Conjoint Simulations," Sawtooth Software Research Paper Series.
[26] Orme, B. K., and Huber, J., 2000, "Improving the value of conjoint simulations," Marketing Research, **12**(4), pp. 12–20.
[27] Hausman, J. a, and Wise, D. a, 1978, "A Conditional Probit Model for Qualitative Choice: Discrete Decisions Recognizing Interdependence and Heterogeneous Preferences," Econometrica, **46**(2), pp. 403–426.
[28] Revelt, D., and Train, K., 1998, "Mixed Logit with Repeated Choices: Households' Choices of Appliance Efficiency Level," Review of Economics and Statistics, **80**(4), pp. 647–657.
[29] Tu, J., Choi, K. K., and Park, Y. H., 1999, "A New Study on Reliability-Based Design Optimization," Journal of Mechanical Design, **121**(4), pp. 557–564.
[30] Lee, I., Choi, K. K., Du, L., and Gorsich, D., 2008, "Dimension reduction method for reliability-based robust design optimization," Computers & Structures, **86**(13), pp. 1550–1562.
[31] Lee, I., Shin, J., and Choi, K. K., 2013, "Equivalent target probability of failure to convert high-reliability model to low-reliability model for efficiency of sampling-based RBDO," Structural and Multidisciplinary Optimization, **48**(2), pp. 235–248.
[32] Du, X., and Chen, W., 2000, "Towards a Better Understanding of Modeling Feasibility Robustness in Engineering Design," Journal of Mechanical Design, **122**(4), p. 385.
[33] Chiralaksanakul, A., and Mahadevan, S., 2005, "First-Order Approximation Methods in Reliability-Based Design Optimization," Journal of Mechanical Design, **127**(5), pp. 851–857.
[34] Shin, J., and Lee, I., 2014, "Reliability-Based Vehicle Safety Assessment and Design Optimization of Roadway Radius and Speed Limit in Windy Environments," Journal of Mechanical Design, **136**(8), p. 081006.
[35] Lee, I., Noh, Y., and Yoo, D., 2012, "A Novel Second-Order Reliability Method (SORM) Using Noncentral or Generalized Chi-Squared Distributions," Journal of Mechanical Design, **134**(10), p. 100912.
[36] Du, X., and Chen, W., 2004, "Sequential Optimization and Reliability Assessment Method for Efficient Probabilistic Design," Journal of Mechanical Design, **126**(2), pp. 225–233.
[37] Kalsi, M., Hacker, K., and Lewis, K., 2001, "A Comprehensive Robust Design Approach for Decision Trade-Offs in Complex Systems Design," Journal of Mechanical Design, **123**(1), pp. 1–10.
[38] Phadke, M. S., 1995, Quality engineering using robust design, Prentice Hall PTR.
[39] Chen, W., Allen, J. K., Tsui, K.-L., and Mistree, F., 1996, "A Procedure for Robust Design: Minimizing Variations Caused by Noise Factors and Control Factors," Journal of Mechanical Design, **118**(4), pp. 478–485.
[40] See, T.-K., Gurnani, A., and Lewis, K., 2004, "Multi-Attribute Decision Making Using Hypothetical Equivalents and Inequivalents," Journal of Mechanical Design, **126**(6), pp. 950–958.
[41] Sawtooth Software inc., 2011, "Sawtooth Software SSI Web 7.0."
[42] Feller, W., 2008, An introduction to probability theory and its applications, John Wiley & Sons.
[43] Sullivan, E., Ferguson, S., and Donndelinger, J., 2011, "Exploring differences in preference heterogeneity representation and their influence in product family design," ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (IDETC/CIE2011), Paper No. DETC2011-48596.
[44] Sawtooth Software inc., 2007, "Sawtooth Software Latent Class 4.0.8."
[45] The Mathworks Inc., 2015, "MATLAB 2015b."
[46] Deb, K., 2001, Multi-objective optimization using evolutionary algorithms, John Wiley & Sons.
[47] Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T., 2002, "A fast and elitist multiobjective genetic algorithm: NSGA-II," IEEE Transactions on Evolutionary Computation, **6**(2), pp. 182–197.
[48] Foster, G., Turner, C., Ferguson, S., and Donndelinger, J., 2014, "Creating targeted initial populations for genetic product searches in heterogeneous markets," Engineering Optimization, **46**(12), pp. 1729–1747.